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ELECTRIC IGNITION FOR AUTOMOBILE ENGINES

BY

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THESIS

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ELECTRIC IGNITION FOR AUTOMOBILE ENGINES

Part I Ignition Circuit

I Introduction

Automobile engineering has made great progress during the past few years. The new devices of yesterday to-day are commonplace.

As for spark ignition, there still exists much discussion about the relative merits of the battery and magneto systems.

The phenomenon of the electric spark was considered by Faraday many years ago. But a perfect analysis of spark has not yet been given.

The object of this thesis is to study the electrical phenomena of the primary and the secondary circuit of ignition apparatus at the instant of sparking.

For mathematical analysis the battery ignition is taken, but we may apply the same theory for magneto ignition by slight modifications.

The latter part of this thesis deals with the heat quantity in a single spark of the various ignition systems.

II Fundamental Principles of Ignition Systems

In developing the theory, the schematic coil shown in Fig.1 will be considered.

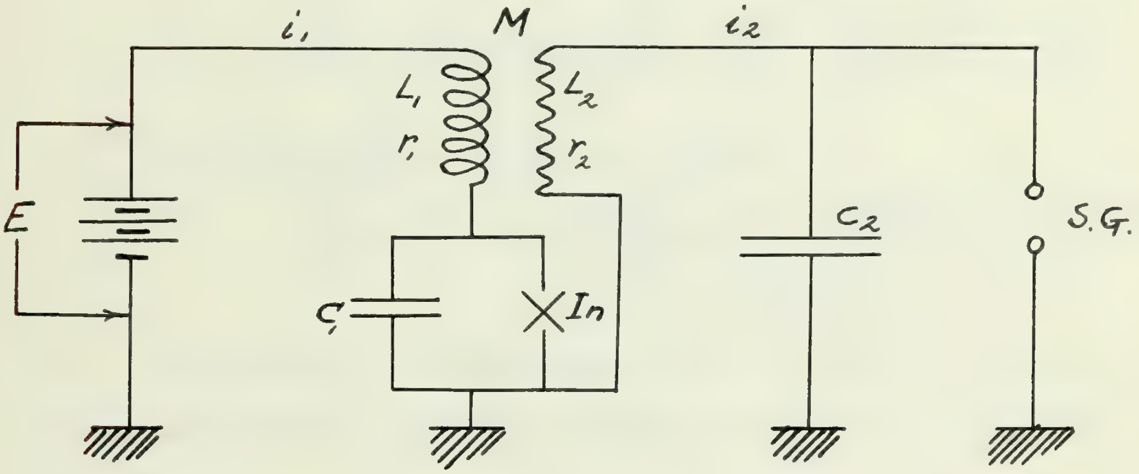


Fig.1

E	Primary terminal voltage
i_1, i_2	Primary and secondary currents
L_1, L_2	Primary and secondary Inductances
r_1, r_2	Primary and secondary resistances
C_1, C_2	Primary and secondary capacities
In	Interrupter
S.G.	Spark Gap
M	Mutual inductance

The fundamental equations of the circuit shown above are:

For the primary circuit,

$$E = r_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + \frac{q_1}{C_1} \qquad (1)$$

For the secondary circuit,

$$0 = r_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + \frac{q_2}{C_2} \qquad (2)$$

where $q_1 = \int i_1\,dt \qquad q_2 = \int i_2\,dt$

The differential equations for the primary and the secondary currents respectively are reduced from equations (1) and (2).

$$(L_1L_2 - M^2)\frac{d^4 i_1}{dt^4} + (r_1L_2 + r_2L_1)\frac{d^3 i_1}{dt^3} + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + r_1r_2\right)\frac{d^2 i_1}{dt^2} + \left(\frac{r_1}{C_2} + \frac{r_2}{C_1}\right)\frac{di_1}{dt} + \frac{i_1}{C_1C_2} = 0 \qquad (3)$$

$$(L_1L_2 - M^2)\frac{d^4 i_2}{dt^4} + (r_1L_2 + r_2L_1)\frac{d^3 i_2}{dt^3} + \left(\frac{L}{C_2} + \frac{L_2}{C_1} + r_1r_2\right)\frac{d^2 i_2}{dt^2} + \left(\frac{r_1}{C_2} + \frac{r_2}{C_1}\right)\frac{di_2}{dt} + \frac{i_2}{C_1C_2} = 0 \qquad (4)$$

(See Appendex I)

The complete integrations of these fourth order differential equations can be accomplished, but is of minor interest.

To avoid the complication, we shall study the phenomena for definite positions of the interrupter and consider the primary and the secondary resistance, inductance, and capacity constant in most cases.

III Phenomena of closing the interrupter

To speak rigorously, a secondary charging current exists in the secondary circuit in this case, but we shall neglect this current.

No voltage is impressed on the primary condenser, because the interrupter is closed. So $M\frac{di_2}{dt}$ and $\frac{\int i_1 dt}{C_1}$ are both zero.

The equation (1) page3 becomes

$$E = r_1 i_1' + L_1 \frac{di_1}{dt} \qquad (5)$$

$$\therefore \frac{E}{L_1} = \frac{di_1}{dt} + \frac{r_1 i_1'}{L_1}$$

$$i_1' = e^{-\frac{r_1}{L_1}t}\left[\int e^{\frac{r_1}{L_1}t}\frac{E}{L_1}dt + C\right]$$

$$= \frac{E}{r_1} + C e^{-\frac{r_1}{L_1}t}$$

When $t = 0$, $i_1 = 0$

$$0 = \frac{E}{r_1} + C \qquad \therefore C = -\frac{E}{r_1}$$

$$\text{So,} \quad i_1' = \frac{E}{r_1}\left(1 - e^{-\frac{r_1}{L_1}t}\right) \qquad (6)$$

which is known as the Helmholz formula.

When $t = \infty$,

$$i_1' = I = \frac{E}{r_1}$$

$$\therefore I \propto \frac{1}{r_1}$$

The instantanious primary voltage

$$e_1 = L_1 \frac{di_1}{dt} = + L_1 \left(+ \frac{r_1}{L_1} e^{-\frac{r_1}{L_1} t} \right) \frac{E}{r_1}$$

$$= E e^{-\frac{r_1}{L_1} t}$$

The following numerical constants are given for the Connecticut Igniter Model-16 :

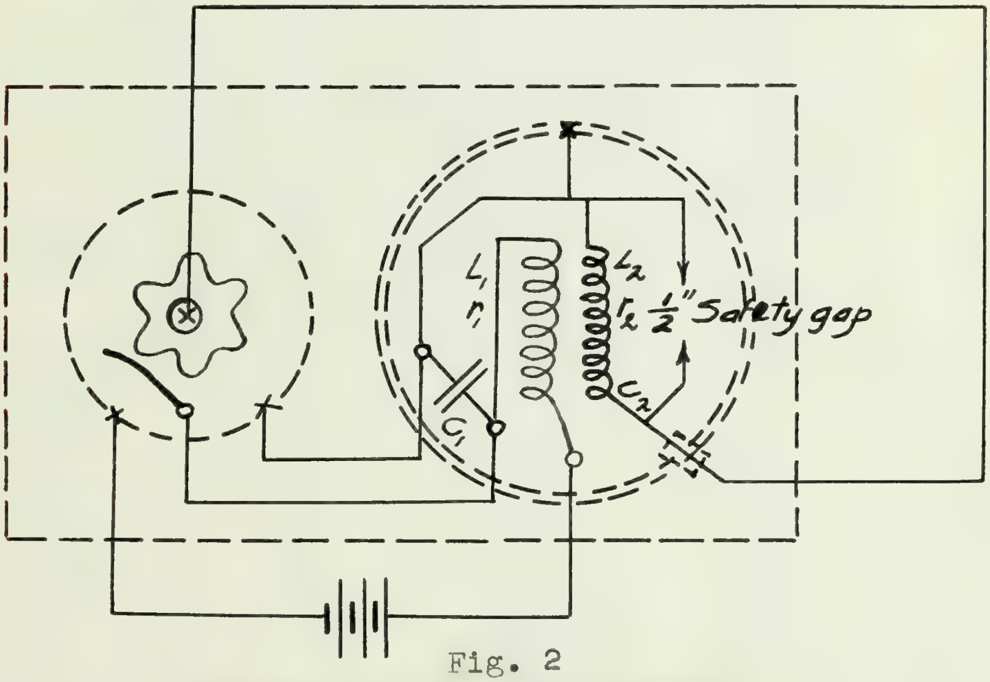
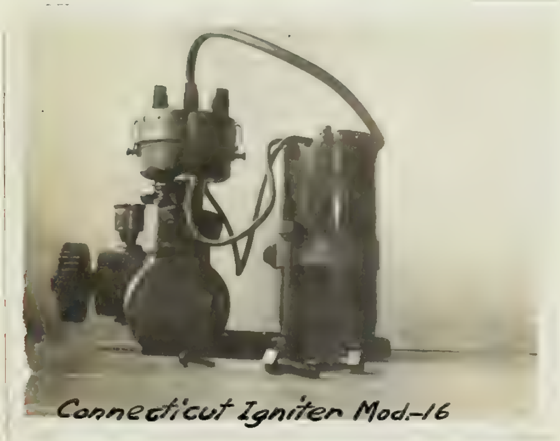


Fig. 2



Connecticut Igniten Mod.-16

L_1	.011 henry			
r_1	1.1 ohms		r_2	10,000 ohms
C_1	.25 micro-farad		C_2	smaller than .0001m.f.

The relation between time and the primary current culculated by equation(6) is shown in the following table.

Table 1 Growth of the primary current

I 5.2 amperes

r_1 1.1 ohms

L_1 .011 henry

So, $E = I r = 5.725$ volts

$$\frac{r_1}{L_1} = \frac{1.1}{.011} = 100$$

$$\log e = \log 2.72 = .434$$

t sec.	$\frac{r_1}{L_1}t$	$e^{-\frac{r}{L}t}$	$1-e^{-\frac{r}{L}t}$	i_1 ampere
0	0	1	0	0
.002	.2	.834	.166	.87
.004	.4	.664	.336	1.75
.006	.6	.542	.458	2.38
.008	.8	.441	.559	2.91
.01	1	.368	.632	3.29
.012	1.2	.293	.707	3.68
.014	1.4	.238	.762	3.96
.016	1.6	.195	.805	4.19
.018	1.8	.158	.842	4.38
.02	2	.129	.371	4.53
.022	2.2	.105	.895	4.66
.024	2.4	.086	.814	4.75
.03	3	.045	.955	4.96
.035	3.5	.025	.972	5.05
.04	4	.017	.983	5.11
.045	4.5	0	1	5.2

The curve corresponding to Table 1 is shown in Fig.6, page 13. Its oscillogram which is taken by author is seen below.

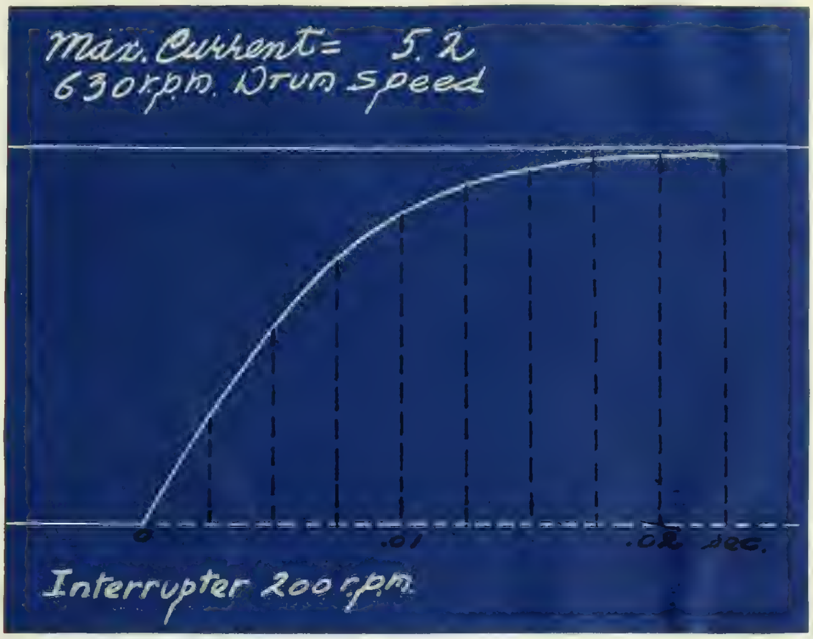


Fig. 3

If we enlarge this oscillogram and plot a curve as in Fig.6, page 13, there is seen a certain difference from Curve A. Probably this difference depends upon the fact that magnetization curve of iron is not a straight line and con-consequently the primary inductance is not constant.

We will now consider the case when the primary inductance is not constant.

The values of the following table are taken from the oscillogram.

Table 2

t sec.	i, ampere
0	0
.0025	1.39
.005	2.6

.0075	3.54
.01	4.18
.0125	4.61
.015	4.8
.0175	4.96
.02	5.04
.0225	5.07

Closing the Interrupter in the Case of Variable Inductance:

In the former discussion , we assumed that the inductance L, is constant, while it is variable with any circuit which contains an iron core. The variation of inductance depends upon the magnitude of the air gap and the quality of the iron.

An example of the saturation curve of an entirely closed ferro-magnetic circuit is shown in Fig.(4).

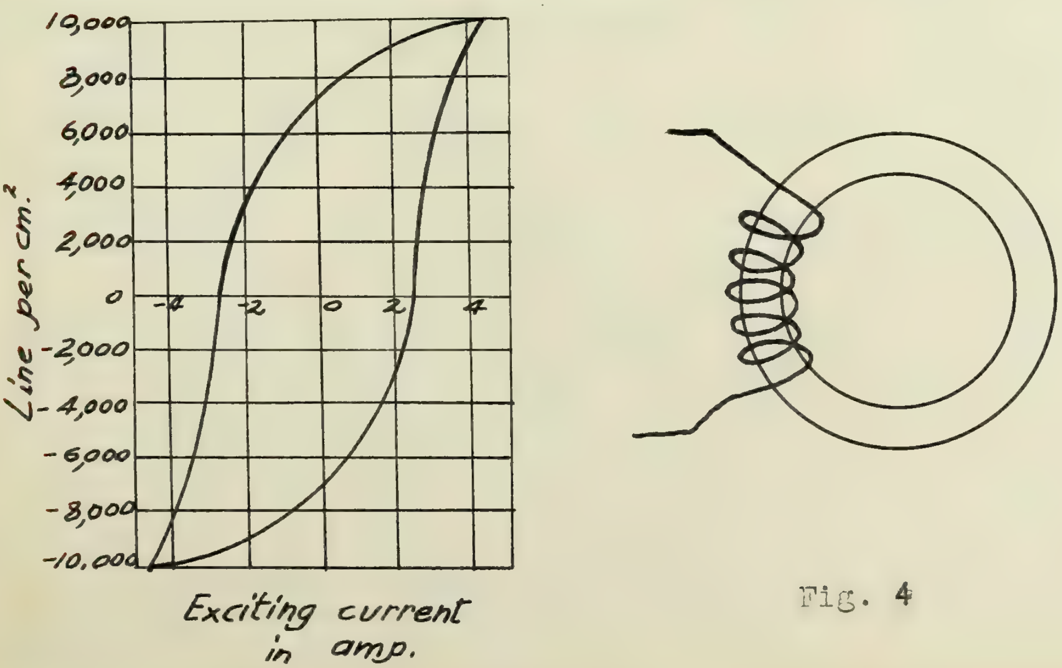


Fig. 4

If the magnetic circuit consists partly of air and partly of iron, the saturation curve is modified by the influence of air gap.

When the air gap is very small, the corresponding curves are a and b, Fig.5. If the gap becomes larger, the saturation curve approaches the dotted line, and the hysteresis loop vanishes.

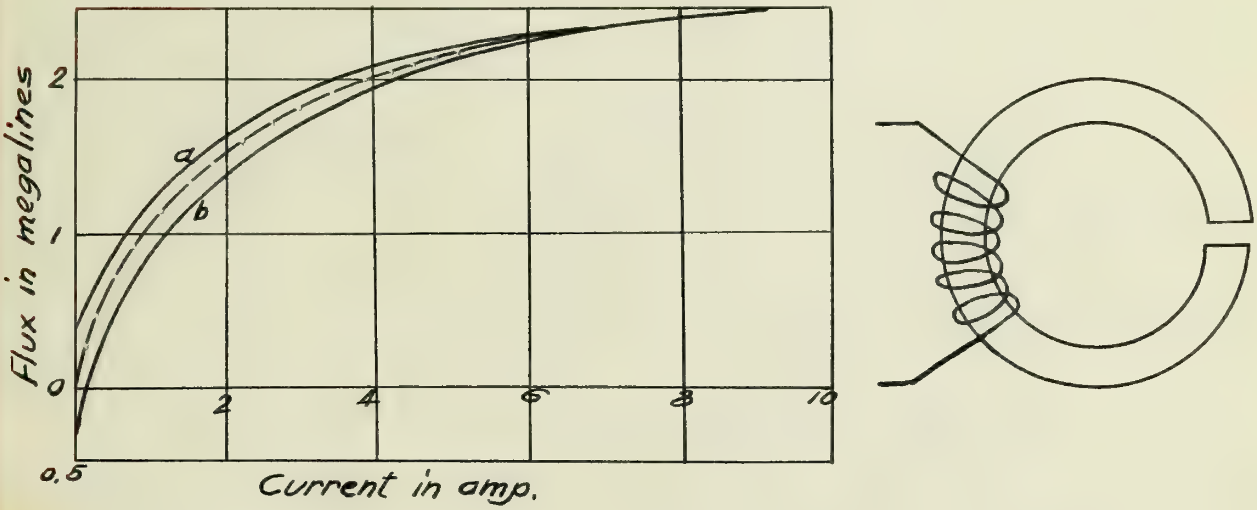


Fig. 5

Frolich's equation for such a saturation curve, which is modified by Kennelly, is shown below.

$$\phi = \frac{k_1 i}{1 + ki}$$ where ϕ = flux (7)

While $$L = \frac{N\phi}{i\,10^8}$$ where N : number of turns

$$\therefore L = \frac{N k_1 10^{-8}}{1 + ki}$$

If we substitute $\frac{N k_1}{10^8}$ for λ,

$$L = \frac{\lambda}{1 + ki}$$

The equation (1), page 3 becomes

$$E = ir + \frac{d}{dt}(L, i) \qquad (8)$$

$$= ir + L\frac{di}{dt} + i\frac{dL}{dt}$$

$$= ir + \frac{\lambda}{1+ki}\frac{di}{dt} - \frac{i\lambda k}{(1+ki)^2}\frac{di}{dt}$$

The variables are separable in the above equation.

So,

$$\frac{di}{(E-ir)(1+ki)^2} = \frac{dt}{\lambda}$$

$$\therefore \int \frac{di}{(E-ir)(1+ki)^2} = \frac{t}{\lambda} + c$$

$$\int \frac{di}{(E-ir)(1+ki)^2} = \int \frac{r^2}{(E-ir)a^2}\,di + \int \frac{kr}{(1+ki)a^2}\,di$$

$$+ \int \frac{k}{(1+ki)^2 a}\,di \qquad \text{(see Appendex 2)}$$

$$= -\frac{r}{a^2}\log(E-ir) + \frac{r}{a^2}\log(1+ki) - \frac{1}{a} \times \frac{1}{1+ki}$$

$$= \frac{1}{a}\left(\frac{r}{a}\log\frac{1+ki}{E-ir} - \frac{1}{1+ki}\right]$$

$$\therefore \quad \frac{t}{\lambda} + C = \frac{1}{a}\left(\frac{r}{a}\log\frac{1+ki}{E-ir} - \frac{1}{1+ki}\right]$$

where a = r + Ek.

When t = 0 & i = 0 ,

$$\therefore \quad C = \frac{1}{a}\left(\frac{r}{a}\log\frac{1}{E} - 1\right)$$

$$\therefore \quad t = \frac{\lambda}{a}\left(\frac{r}{a}\,\mathrm{Log}_E\,\frac{1+ki}{E-ir} + \frac{ki}{1+ki}\right) \qquad (9)$$

(Berg's Advanced Course of Electrical Engineering)

The variable inductance for the corresponding values of exciting current i is measured for a certain coil which has an open iron core. Then, we may find the constants k and λ. Using these constants, we will study how the current builds up in the coil considered in the previous article.

By the equation (9),

$$t = \frac{\lambda}{a}\left\{\frac{r}{a}\log E\,\frac{1+ki}{E-ir} + \frac{ki}{1+ki}\right\}$$

If	k	.1128	E	5.725
	λ	.01018	r_1	1.1

a = r + Ek = 1.1 + 5.725 x .1128

= 1.745

So, the above equation becomes

$$t = \frac{.01018}{1.745}\left\{\frac{11}{1.745} \times \frac{1}{.434}\log 5.725 \times \frac{1+.1128i}{5.725-1.1i} + \frac{.1128i}{1+.1128i}\right\}$$

$$\therefore\ t = .00584 \times \left\{1.45\log\frac{5.725(1+.1128i)}{5.725-1.1i} + \frac{.1128i}{1+.1128i}\right\} \quad (10)$$

Table 3 is calculated by the equation (10).

Table 3

i ampere	$A=\frac{r}{a}\log E\frac{1+ki}{E-ir}$	$B=\frac{ki}{1+ki}$	A+ B	t sec.
0	0	0	0	0
1	.202	.1915	.302	.00176
2	.433	.1815	.614	.00358
3	.725	.253	.977	.0057
4	1.15	.309	1.46	.00853
4.5	1.52	.337	1.86	.01087
5	2.32	.36	2.68	.0157

The corresponding curve to this table is shown in Fig. 6, page 13.

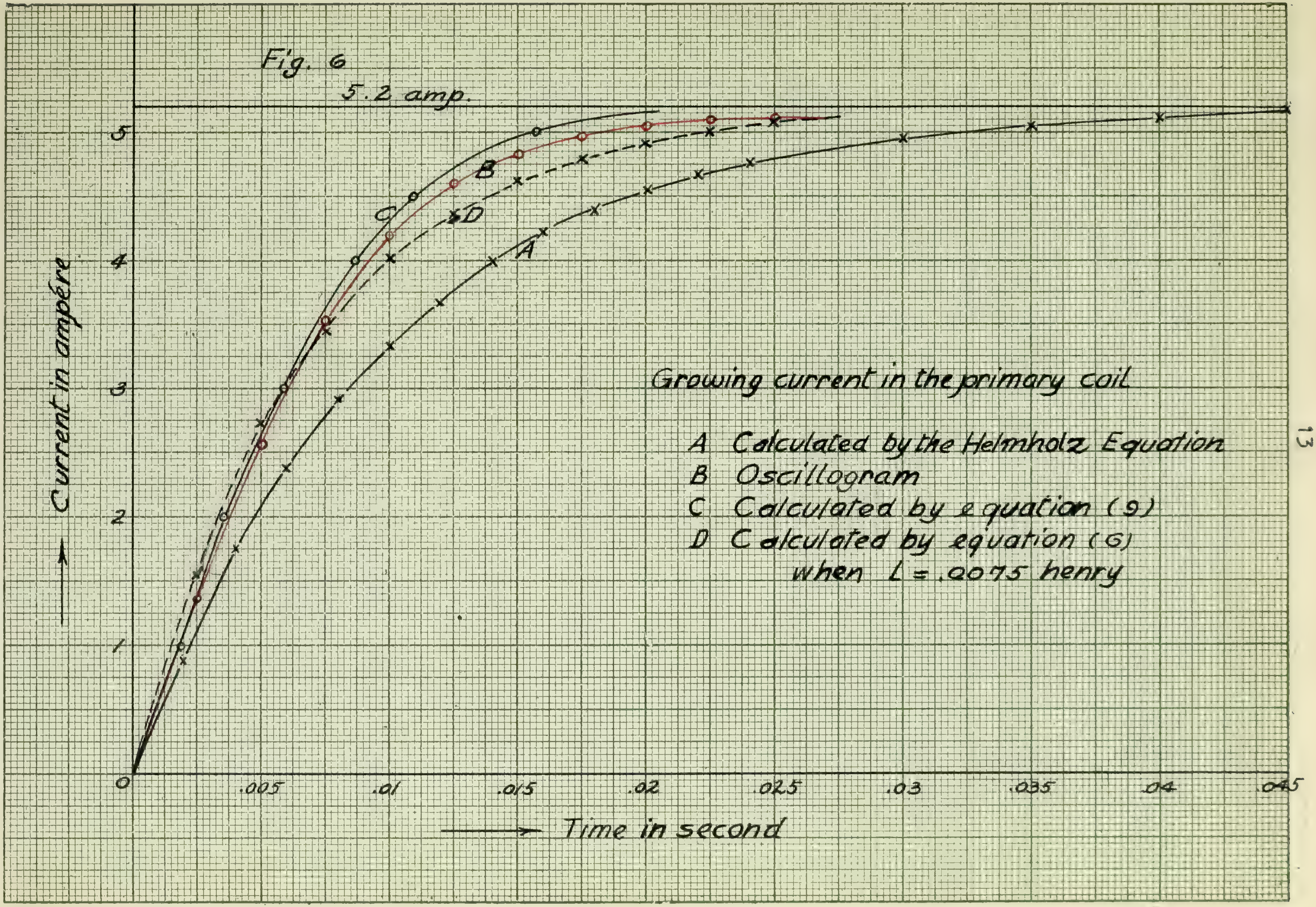


Fig. 6

Growing current in the primary coil

A Calculated by the Helmholz Equation
B Oscillogram
C Calculated by equation (9)
D Calculated by equation (6) when L = .0075 henry

Comparing the three cases in Fig.6, it is seen that curve C which is calculated by equation (9) is nearer to the oscillogram than the curve A calculated by equation (6). This means that the coefficient of self-induction is changing with the saturation of the magnetic core.

It is interesting to notice that the time to build the primary current is a function of the primary resistance as well as of the primary inductance. If the resistance is higher, the time is shorter, and if the inductance is smaller, the time is shorter. But, the resistance should not be too large, because then the primary final current is reduced. The inductance should not be too small, because then the secondary peak voltage will be largely reduced in the oscillating discharge.

If we make the inductance smaller, it is seen that the time to build the current is largely reduced as in curve D, Fig. 6.

From the above discussion the following conclusions may be drawn :

I The time to establish the primry current is short when L_1 is small or when r_1 is large.

II With a closed iron core the inductance L_1 decreases as the magnetizing current inccreases. Hence the so-called time constant $\frac{r_1}{L_1}$ becomes larger for higher magnetic saturation.

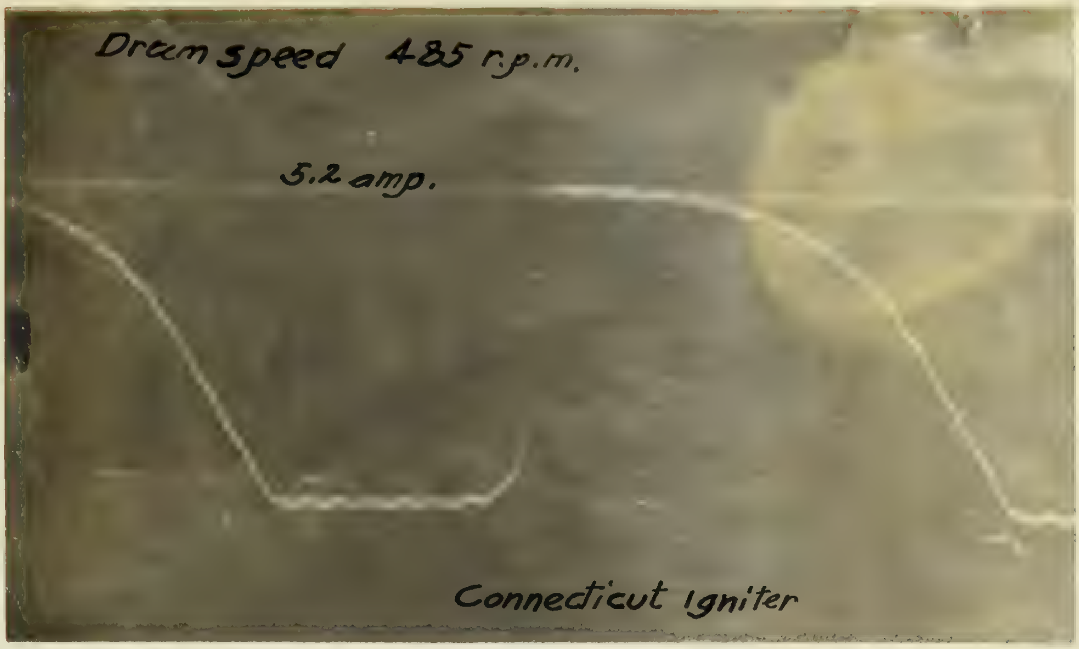


Fig. 7

The ripples in the current wave seem to be caused by the free oscillation of the mirror of the oscillograph.

IV Phenomena of Opening the Interrupter

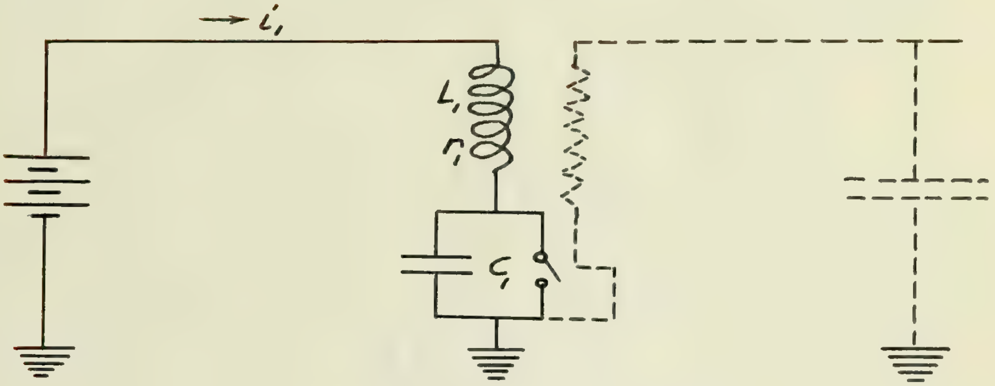


Fig. 8

We assume that the charging current in the secondary is negligible. Then equation (1) becomes

$$E = r_1 i_1 + L_1 \frac{di_1}{dt} + \frac{\int i_1 dt}{C_1} \qquad (11)$$

If we differentiate this equation with respect to t,

$$L_1 \frac{d^2 i_1}{dt^2} + r_1 \frac{di_1}{dt} + \frac{i_1}{C_1} = 0$$

or

$$\frac{d^2 i_1}{dt^2} + \frac{r_1}{L_1} \frac{di_1}{dt} + \frac{i_1}{L_1 C_1} = 0$$

The general solution of this equation is

$$i_1 = A_1 e^{m_1 t} + A_2 e^{m_2 t}$$

where m_1 and m_2 are the roots of the auxiliary equation.

$$m^2 + \frac{r_1}{L_1} m + \frac{1}{C_1 L_1} = 0$$

So, $$m_1 = -\frac{r_1}{2L_1} + \sqrt{\frac{r_1^2}{4L_1^2} - \frac{1}{C_1L_1}} = -\alpha + \beta$$

$$m_2 = -\frac{r_1}{2L_1} - \sqrt{\frac{r_1^2}{4L_1^2} - \frac{1}{C_1L_1}} = -\alpha - \beta$$

where $\alpha = \frac{r_1}{2L_1}$, and $\beta = \sqrt{\frac{r_1^2}{4L_1^2} - \frac{1}{C_1L_1}} = \frac{1}{2L_1}\sqrt{r_1^2 - \frac{2L_1}{C_1}}$

$$\therefore \quad i_1 = A_1 e^{(-\alpha+\beta)t} + A_2 e^{(-\alpha-\beta)t}$$

$$= e^{-\alpha}\{A_1 e^{+\beta t} + A_2 e^{-\beta t}]$$

In the above equation, three conditions are possible:

(a) $$r_1^2 - \frac{4L_1}{C_1} > 0$$

(b) $$r_1^2 - \frac{4L_1}{C_1} < 0$$

(c) $$r_1^2 - \frac{4L_1}{C_1} = 0$$

$\frac{4L_1}{C_1}$ is far larger than r_1^2 with the primary coil under the discussion. So it corresponds to case (b).

$$\frac{1}{2L_1}\sqrt{r_1^2 - \frac{4L_1}{C_1}} = j\frac{1}{2L_1}\sqrt{\frac{4L_1}{C_1} - r^2} = j\beta$$

$$\therefore \quad i_1 = A_1 e^{m_1 t} + A_2 e^{m_2 t}$$

$$= e^{-\alpha t}(A_1 e^{j\beta t} + A_2 e^{j\beta t})$$

$$= e^{-\alpha t}(A \cos \beta t + B \sin \beta t) \qquad (12)$$

where A and B are integration constants.

The electro-motive force across the condencer is,

$$e_c = E - i_1 r_1 - L_1 \frac{di_1}{dt}$$

$$= E - r_1 e^{-\alpha t}(A \cos\beta t + B \sin\beta t) - L_1\{-\alpha e^{-\alpha t}(A \cos\beta t + B \sin\beta t) + \beta e^{-\alpha t}(-A \sin\beta t + B \cos\beta t)]$$

$$\therefore \quad e_c = E - e^{-\alpha t}\left[r_1(A\cos\beta t + B\sin\beta t) - L_1\alpha(A\cos\beta t + B\sin\beta t)\right.$$

$$\left. + \beta L_1(-A\sin\beta t + B\cos\beta t)\right] \qquad (13)$$

When $t = 0$, $i_1 = I$ where I is the value of the current at the moment of opening the interrupter.

From equation (12),

$$I = A$$

From equation (13),

$$0 = E - r_1 I + L_1 I\alpha + L_1 B\beta$$

$$E = I_0 r_1 \qquad \therefore \quad B = \frac{(I_0 - I) r_1 + L_1 I\alpha}{\beta L_1} = \left(\frac{E}{\beta L_1} - \frac{\alpha}{\beta}\right)$$

So, equation (12) becomes

$$i_1 = e^{-\alpha t}\left\{I\cos\beta t + \left(\frac{E}{\beta L_1} - \frac{\alpha}{\beta}\right)\sin\beta t\right\} \qquad (14)$$

and equation (13) becomes

$$e_c = E - e^{-\alpha t}\left[\left\{\frac{I r_1}{2\beta L_1} - \frac{(\alpha^2 + \beta^2) I L_1}{\beta}\right\}\sin\beta t + I r_1\cos\beta t\right] \qquad (15)$$

(see Appendex III)

If E is negligible,

$$e_c = -i_1 r_1 - L_1\frac{di_1}{dt}$$

$$= -e^{-\alpha t}\left[r_1(A\cos\beta t + B\sin\beta t) - L_1\alpha(A\cos\beta t + B\sin\beta t)\right.$$

$$\left. + \beta L_1(-A\sin\beta t + B\cos\beta t)\right]$$

$$i_1 = e^{-\alpha t}(A\cos\beta t + B\sin\beta t)$$

When $t = 0$, $i_1 = I$.

$$\therefore \quad I = A \quad \text{and} \quad B = -\frac{\alpha}{\beta} I$$

$$\therefore \quad i = I e^{-\alpha t}(\cos\beta t - \frac{\alpha}{\beta}\sin\beta t) \qquad (16)$$

and $$e_c = -e^{-\alpha t}(I r_1 \cos\beta t + \frac{I\alpha r_1}{\beta}\sin\beta t - I L_1 \alpha\cos\beta t$$

$$+ I\alpha^2 L_1 \frac{1}{\beta}\sin\beta t - L_1\beta I\sin\beta t - I\alpha L_1\cos\beta t)$$

$$= -e^{-\alpha t}(\frac{-I\alpha r_1}{\beta} + \frac{I\alpha r_1}{2\beta} - L_1\beta I)\sin\beta t$$

$$= I e^{-\alpha t}(\frac{\frac{r_1^2}{2L_1} + \frac{2}{C_1} - \frac{r^2}{2L_1}}{2\beta})\sin\beta t$$

$$= I e^{-\alpha t}\frac{1}{C_1\beta}\sin\beta t \qquad (17)$$

Equations (16) and (17) are reduced by R.Colly in the Wiedemann Annalen, vol.44, page 109.

$\frac{\alpha}{\beta}$ in equation (16) is very small in our case.

So, $$i_1' = I e^{-\alpha t}\cos\beta t$$

$$e_c = I e^{-\alpha t}\frac{1}{C_1\beta}\sin\beta t$$

It is evident that the current and voltage are oscillatory and damped.

Their period of oscillation is

$$T = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\frac{1}{L_1C_1} - \frac{r_1^2}{4L_1^2}}}$$

The damping coefficient is equal to $\alpha = \frac{r_1}{2L_1}$.

$$\beta \doteqdot \frac{1}{\sqrt{L_1C_1}}$$

$$\therefore \quad T = 2\pi\sqrt{L_1C_1}$$

$$\therefore \quad i_1 = I e^{-\alpha t}\cos\frac{1}{\sqrt{L_1C_1}}t \qquad (18)$$

$$e_c = I e^{-\alpha t}\sqrt{\frac{L_1}{C_1}}\sin\frac{1}{\sqrt{L_1C_1}} \qquad (19)$$

If there is some magnetic leakage in the primary and secondary,

$$L_1 = A_1 n_1^2$$

$$L_2 = A_2 n_2^2$$

$$M = A_3 n_1 n_2$$

where n_1, n_2 are number of primary and secondary turns.

If there is no leakage between primary and secondary,

$$A_1 = A_2 = A_3$$

$$M = L_1 L_2$$

$$E_{1\,max.} = \sqrt{\frac{L_1}{C_1}}\, I \qquad \text{from equation (19)}$$

and $$E_{2\,max.} = \frac{M}{\sqrt{L_1 C_1}}\, I = \sqrt{\frac{L_2}{C_1}}\, I$$

because the secondary induced e.m.f. is $-M\frac{di}{dt} = I \sin \frac{1}{\sqrt{L_1 C_1}} t$, neglecting the damping effect.

From the above discussion, we may conclude :

I The secondary e.m.f. is proportional to the primary current at the moment of rupture.

II $$\frac{E_{2\,max.}}{E_{1\,max.}} = \frac{M}{L_1} = \sqrt{\frac{L_2}{L_1}} = \frac{n_2}{n_1}$$

if there is no leakage flux.

Taking as example an ignition coil which has following constants.

L_1	.011	hënry	r_1	1.1[illegible]	ohms
C_1	20	m.f.	E	5.725	volts
I_0	5.2	amperes			

$$\therefore\ \alpha = \frac{r_1}{2L_1} = \frac{1.1}{2 \times .011} = 50$$

$$\beta = \frac{1}{2L_1}\sqrt{\frac{4L_1}{C_1} - r_1^2} = \frac{1}{2 \times .011}\sqrt{\frac{4 \times .011}{20 \times 10^{-6}} - 1.1^2}$$

$$= \frac{1}{.022}\sqrt{.022 \times 10^5} = 2100$$

If $I_o = I$, equation (14) becomes

$$i_1' = e^{-\alpha t} I \left(\cos \beta t - \frac{\alpha}{\beta} \sin \beta t\right) \qquad (16)$$

$$i_1' = e^{-50t} \times 5.2 \times (\cos 2100t - 0)$$

$$= 5.2 \times e^{-50t} \cos 2100t$$

Table 4

t sec.	e^{-50t}	cos2100t	i ampere
0	1	1	5.2
.00025	.987	.866	4.44
.0005	.975	.5	2.53
.00075	.962	0	0
.001	.951	-.5	-2.45
.00125	.939	-.866	-4.2
.0015	.927	-1	-4.81
.00175	.917	-.866	-4.13
.002	.904	-.5	-2.35
.00225	.883	0	0

The corresponding curve is shown in Fig.9, page 24.

If $I_o = I$, equation (15) becomes

$$e_c = E - e^{-\alpha t} I_o \left\{ \frac{-(\alpha^2 + \beta^2) L}{\beta} \sin \beta t + r \cos \beta t \right]$$

$$= 5.725 - e^{-50t}(-120 \sin 2100t + 5.72 \cos 2100t)$$

By calculating the terminal voltage at the interrupter for each instant, we may determine how the voltage rises up and down at the instant of interruption.

Table 5

t sec.	e^{-50t}	sin2100t	cos2100t	e volt
0	1	0	1	0
.00025	.987	.5	.866	60
.0005	.975	.866	.5	107.4
.00075	.962	1	0	121
.001	.951	.866	-.5	111
.00125	.939	.5	-.866	66.7
.0015	.927	0	-1	-11.1
.00175	.917	-.5	-.866	-45
.002	.904	-.866	-.5	-89
.00225	.883	-1	0	-100
	....	....	....	
.00375	.825	1	0	105
.00525	.769	-1	0	-86.7
.00675	.714	1	0	91.5
.00825	.662	-1	0	-73.8
.00975	.614	1	0	79.3
.01125	.57	-1	0	-62.7
.01275	.53	1	0	69.2
.01425	.49	-1	0	-53.1
.01575	.45	1	0	59.7

.01725	.422	-1	0	-45
.01875	.393	1	0	52.9
.02025	.362	-1	0	-37.8

The corresponding curve is shown in Fig.9, page 24.

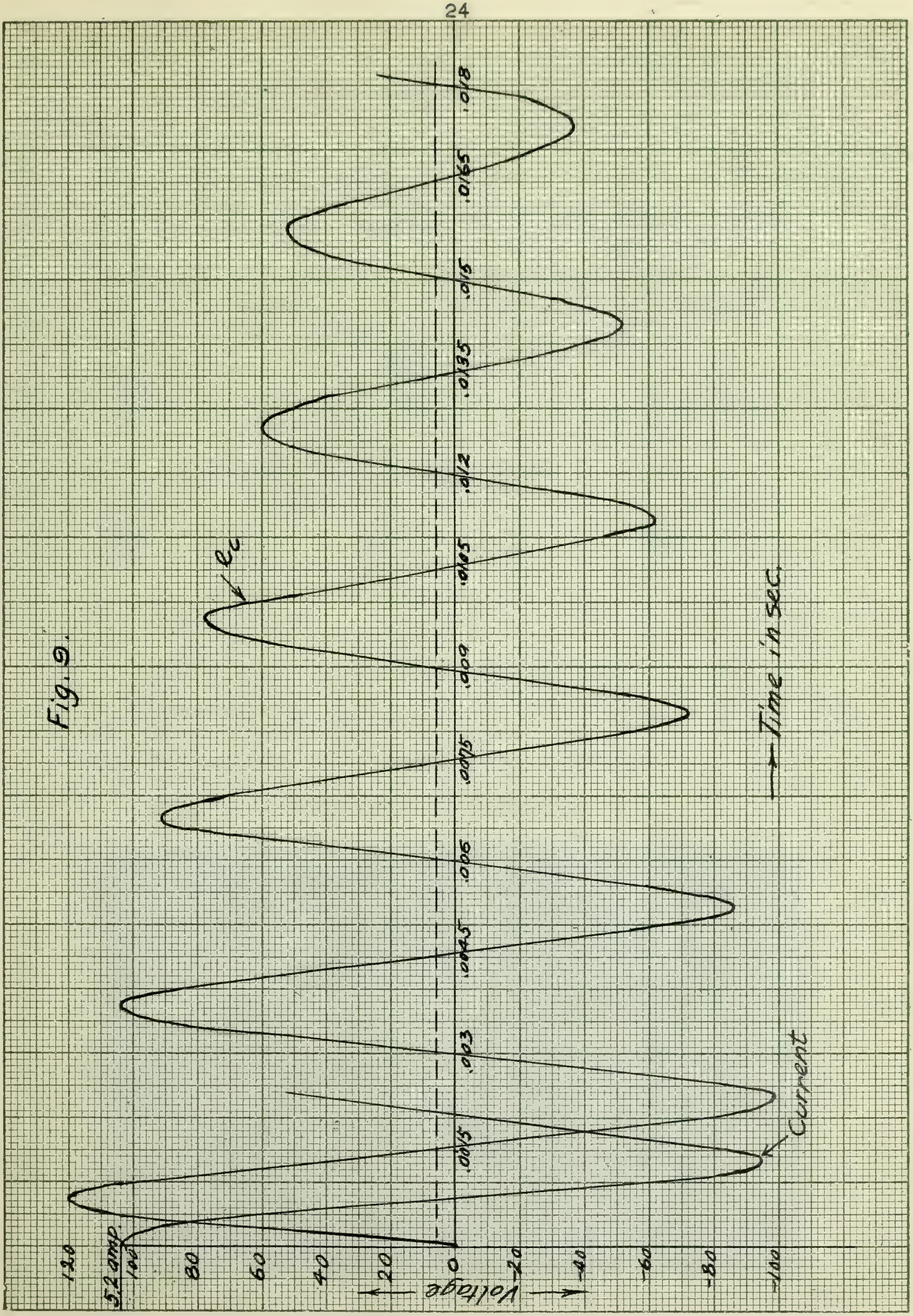


Fig. 9.

120
5.2 amp.
100
80
60
40
20
0
-20
-40
-60
-80
-100
← Voltage →
.0015
.003
.0045
.006
.0075
.009
.0105
.012
.0135
.015
.0165
.018
→ Time in sec.
e_c
Current

An experiment to check this calculation was made by inserting a variable capacity into the interrupter terminals of the Connecticut Igniter.

The connection diagram for the experiment is shown below.

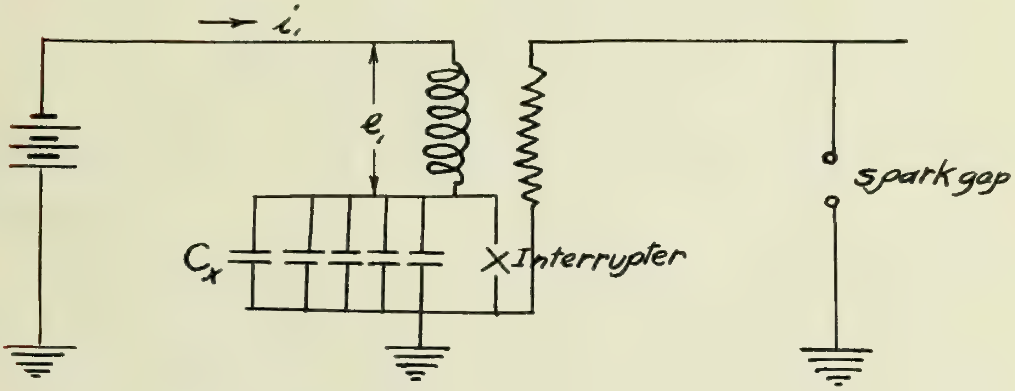
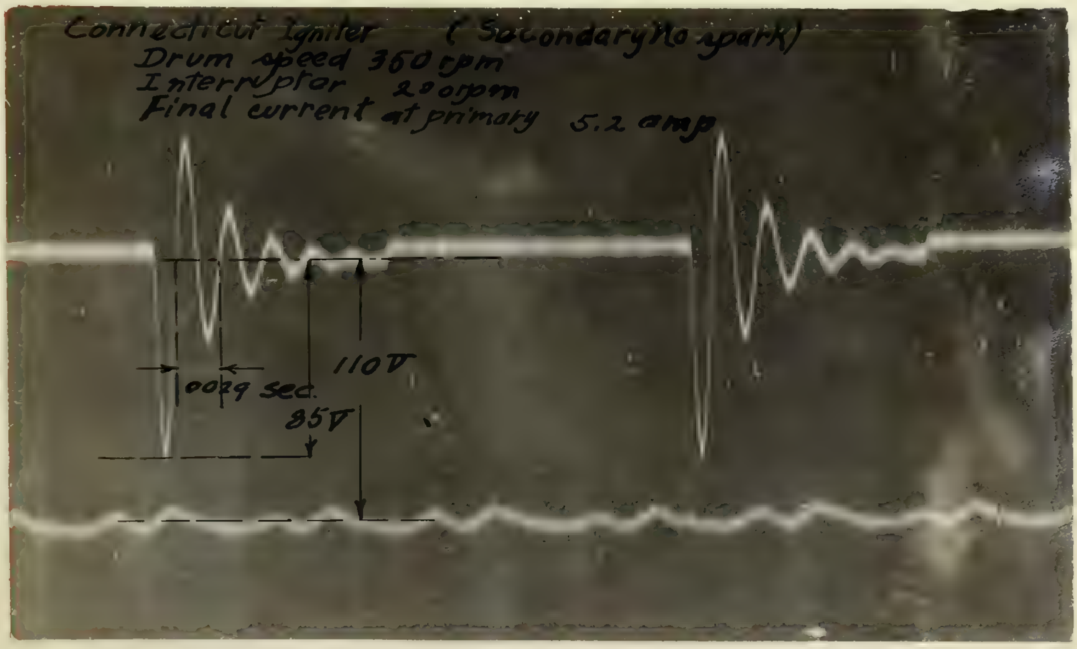


Fig.10

The voltage at the terminals of the primary coil e oscillates, by charging and discharging the current to the condenser C_x after opening the interrupter.

The oscillogram taken when C_x = 20 m.f.



The maximum peak voltage of the oscillogram is about 85 volts.

Comparing this voltage to the voltage wave of Fig.9 which has 120 volts in the maximum of the first oscillation, we may perceive some difference. It is considered that this difference is due to the fact that the energy stored in the core is dissipated by the interrupter spark and is greatly reduced, while we have not considered this in the calculation.

The period of oscillation agrees in the calculation and the experiment. The period is .003 second per cycle by calculation, and the experiment shows us about .0029 second.

It is very interesting to notice that if the capacity of the condenser is increased, the oscillating voltage induced in the primary coil, as well as the secondary, is is lower and the period of oscillation is larger.

The following experimental data shows this fact. The current at the moment of opening the interrupter was always kept constant, that is 5.2 amperes by making the speed same, and C_x in Fig.10 was changed.

Table 7

C_x Condenser capacity	Maximum spark length at secondary
15 m.f.	1.2 m.m.
12	3
9	10

Fig. 11 a

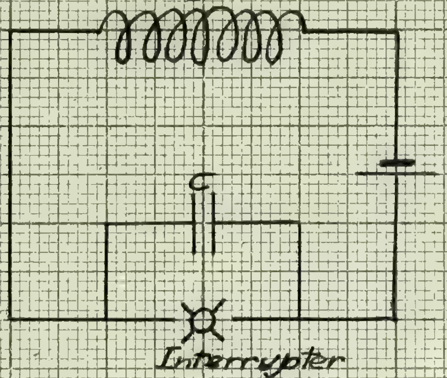
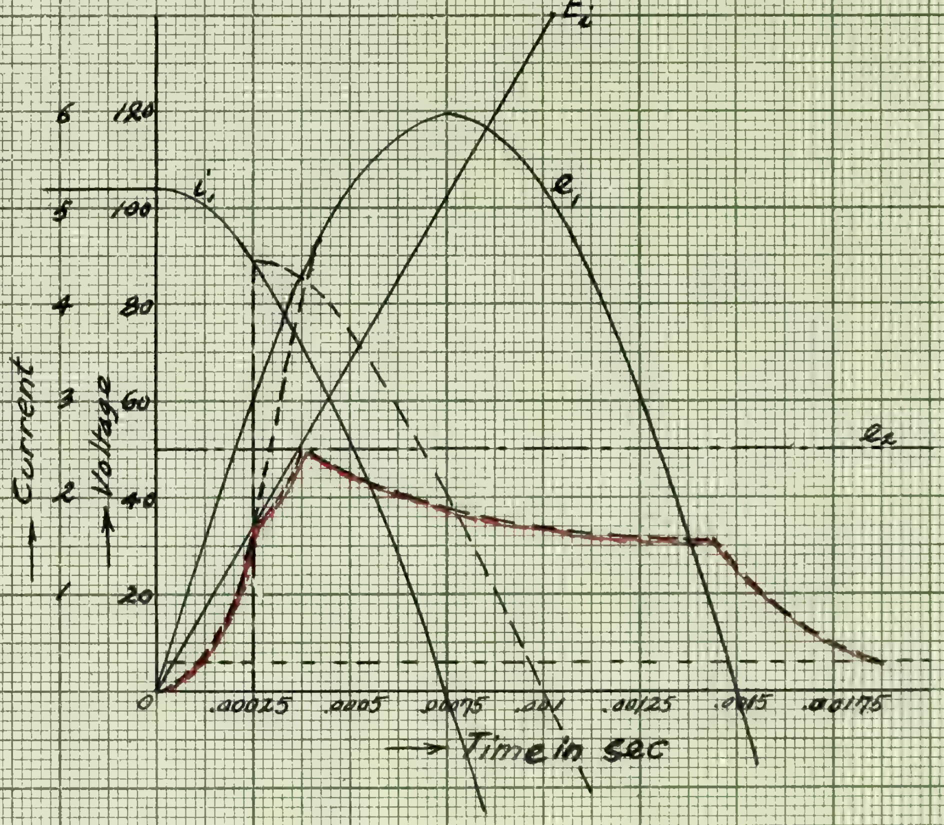


Fig. 11 b

Voltage between interrupter contacts



V Interrupter Spark

We have so far neglected the interrupter spark whereas such a spark always exists with interruption of the primary current. This spark does not occur at the instant when rupture takes place, but a little later. This fact was proved by H.Armagnat in Eclairage electrique (vol.XXII, Jan.27, 1900, page 121).

When we break an inductance circuit which is connected in Fig.11a, the voltage induced at the interrupter terminals does not exactly follow equation (17).

The capacity between the contacts of the interrupter must change from infinity to zero in very short instant. When the capacity is infinite, the voltage curve e_1 in Fig.11 b must be tangent to the absissa. If the contacts open wider, the capacity diminishes gradually and the voltage curve rises along the dotted line.

The disruptive voltage between the contacts E_i' is proportional to the distance between contacts. We assume that it rises proportionally to time.

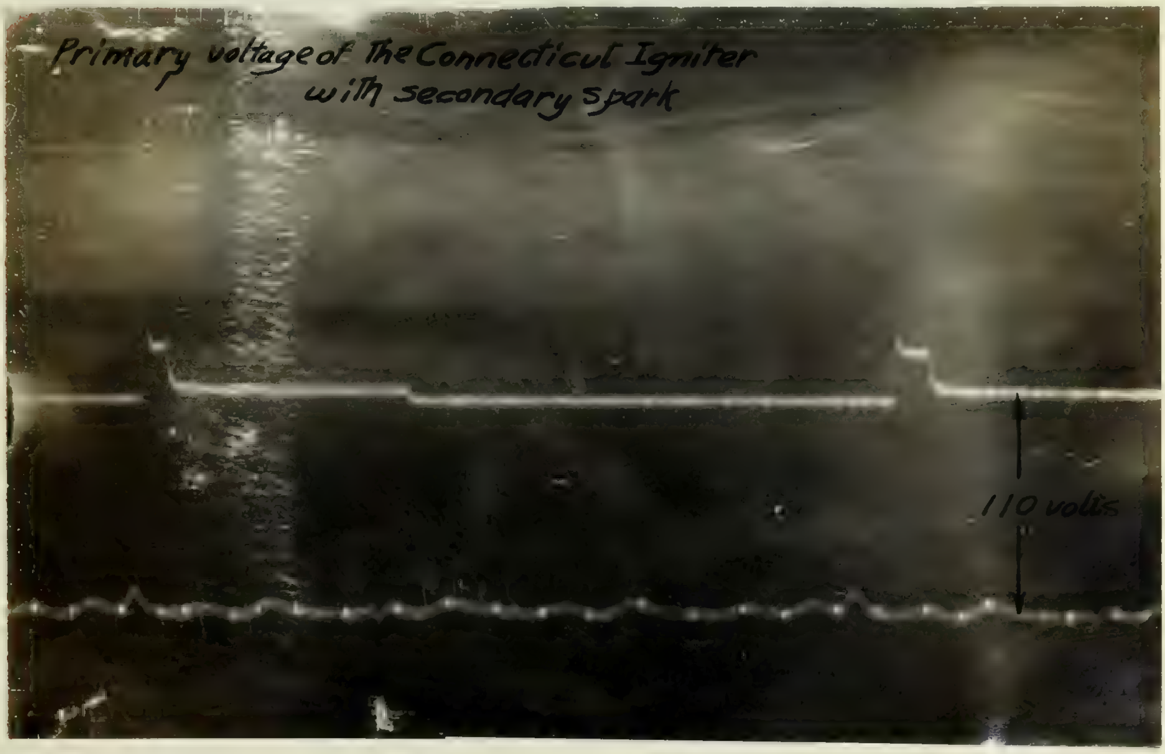
The primary spark starts at the intersection of curves E_i and e_1 , but soon disappears. The terminal voltage will still rise until the secondary spark appears.

The voltage continues as long as the spark exists, and drops down when the ionization of air becomes unstable

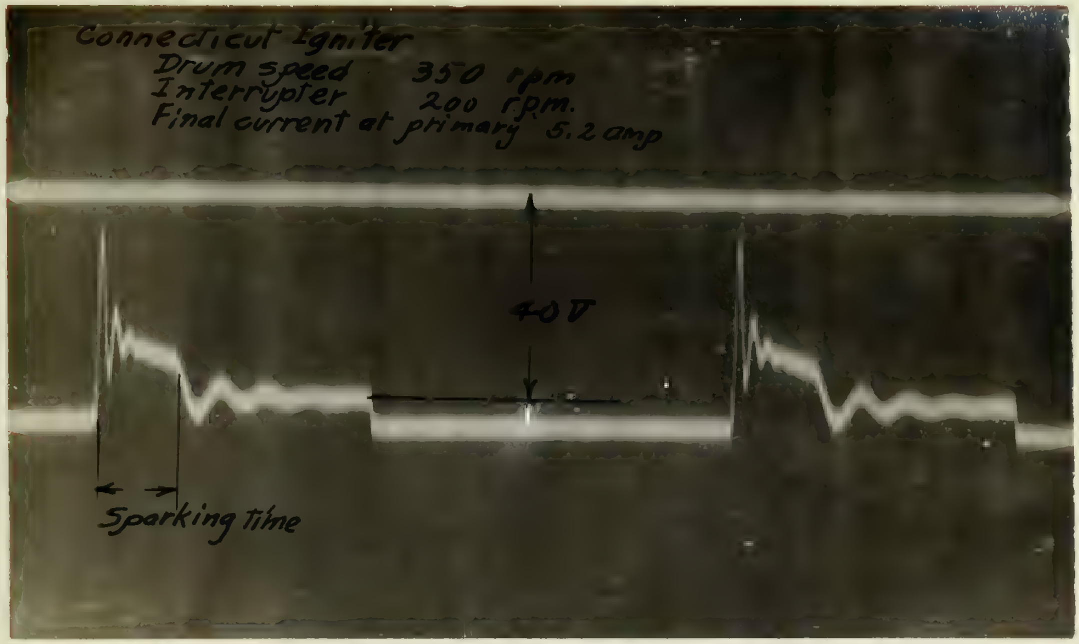
Fig.12(c) shows this phenomenon. The secondary spark starts at A and is sustained a short period of time.

In Fig.12(b), we can not see any spark voltage at the secondary which continues a short period as in (c).

The oscillogram corresponding to Fig.12(c) is shown below.



Voltage variation at the terminals of the primary coil with no additional capacity.



Voltage at the terminal of the primary coil when there is 20 m.f. capacity parallel with the interrupter

If we connect a large capacity in parallel with the interrupter, we see that the duration of the spark is very long, but the disruptive voltage at the secondary is very is very small.

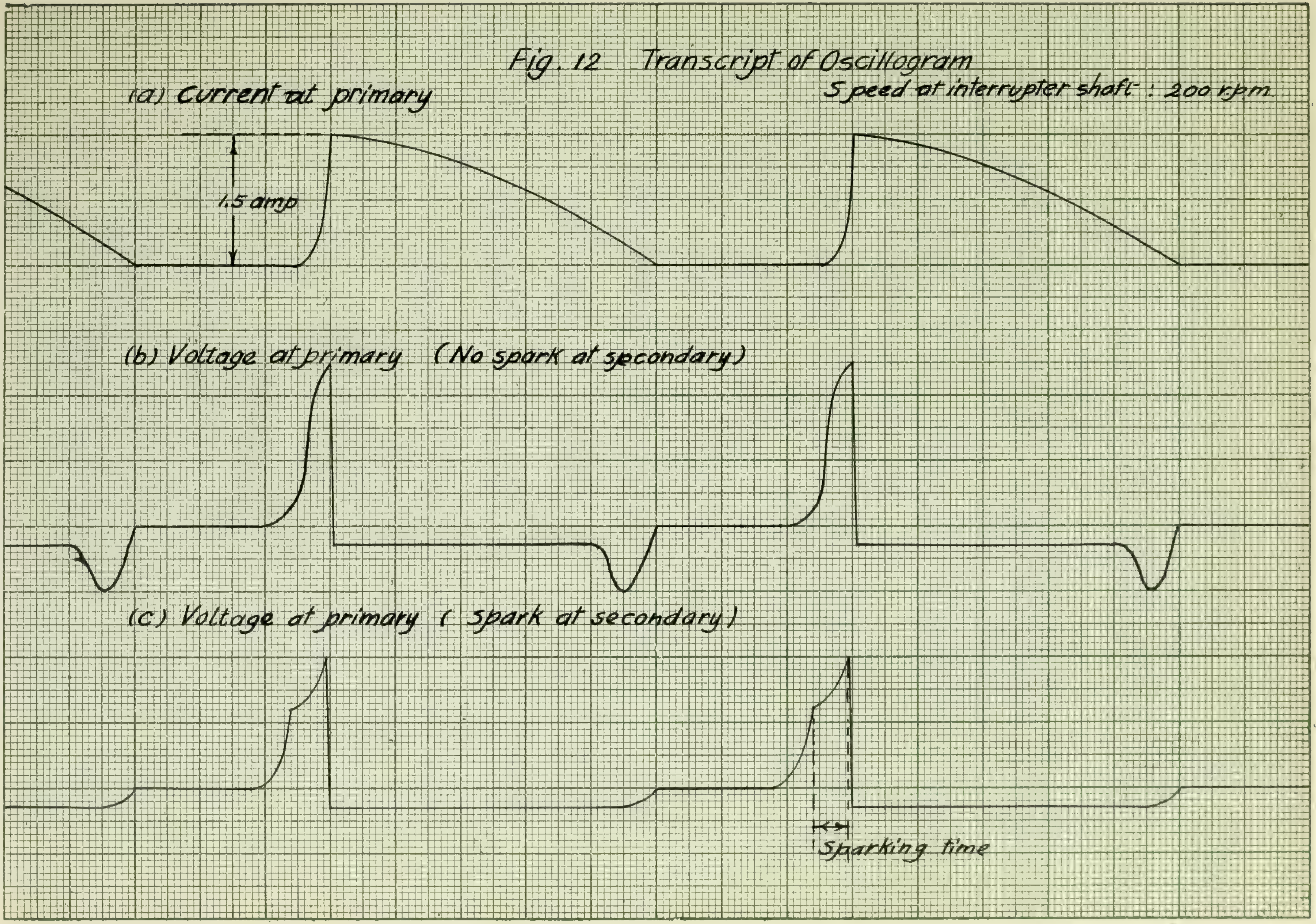


Fig. 12 Transcript of Oscillogram

VI Opening the Interrupter When the Secondary Charging Current is Appreciable

From the oscillographic study, the secondary capacity seems to be negligible. But if we consider this capacity, the developement of the differential equation is as follows:

From equations (1) and (2),

$$\frac{d^2v_1}{dt^2} + \frac{r_1}{L_1}\frac{dv_1}{dt} + \frac{v_1}{L_1C_1} = -\frac{MC_2}{L_1C_1}\frac{d^2v_2}{dt^2} \qquad (a)$$

$$\frac{d^2v_2}{dt^2} + \frac{r_2}{L_2}\frac{dv_2}{dt} + \frac{v_2}{L_2C_2} = -\frac{MC_1}{L_2C_2}\frac{d^2v_1}{dt^2} \qquad (b)$$

$$\therefore \quad \frac{d^2v_1}{dt^2} + 2\delta_1\frac{dv_1}{dt} + (\nu_1^2+\delta_1^2)v_1 = p_{12}\frac{d^2v_2}{dt^2}$$

$$\frac{d^2v_2}{dt^2} + 2\delta_2\frac{dv_2}{dt} + (\nu_2^2+\delta_2^2)v_2 = p_{21}\frac{d^2v_1}{dt^2}$$

where $\delta_1 = \frac{r_1}{2L_1}$, $\nu_1^2+\delta_1^2 = \frac{1}{L_1C_1}$, $p_{12} = -\frac{MC_2}{L_1C_1}$

$\delta_2 = \frac{r_2}{2L_2}$, $\nu_2^2+\delta_2^2 = \frac{1}{L_2C_2}$, $p_{21} = -\frac{MC_1}{L_2C_2}$

The solution of the above simultaneous equations are:

$$v_1 = A\,e^{mt}$$
$$v_2 = B\,e^{mt} \qquad (c)$$

where $m = -\delta \pm i\nu$

From (a), (b), and (c),

$$A\,(x^2+2\delta_1 x + \nu_1^2+\delta_1^2) = p_{12}\,x^2 B \qquad (I)$$

$$B\,(x^2+2\delta_2 x + \nu_2^2+\delta_2^2) = p_{21}\,x^2 A \qquad (II)$$

The solution of equations (a) and (b) is

$$V_1 = A_1 e^{m_1 t} + A_2 e^{m_2 t} + A_3 e^{m_3 t} + A_4 e^{m_4 t}$$

$$V_2 = B_1 e^{m_1 t} + B_2 e^{m_2 t} + B_3 e^{m_3 t} + B_4 e^{m_4 t}$$

$$-\frac{i_1}{C_1} = m_1 A_1 e^{m_1 t} + m_2 A_2 e^{m_2 t} + m_3 A_3 e^{m_3 t} + m_4 A_4 e^{m_4 t}$$

$$-\frac{i_2}{C_2} = m_1 B_1 e^{m_1 t} + m_2 B_2 e^{m_2 t} + m_3 B_3 e^{m_3 t} + m_4 B_4 e^{m_4 t}$$

where m_1 is the four roots of the auxiliary equation.

$m_1, m_2 = -\delta_1 \pm i\nu_1$ $m_3, m_4 = -\delta_2 \pm i\nu_2$

and A and B are constants.

When $t = 0$

$V_1 = 0$ $V_2 = 0$ $i_1 = I$ $i_2 = 0$

$\therefore \ \Sigma A_k = 0$ $\Sigma B_k = 0$

$$\Sigma m_k A_k = -\frac{I}{C_1} = F \qquad \Sigma m_k B_k = 0$$

where k 1,2,3,4

From (II), substituting m for x ,

$$B_k \left(m_k + 2\delta_2 + \frac{\nu_2^2 + \delta_2^2}{m_k} \right) = p_{21} m_k A_k$$

$$\therefore \frac{B_1}{m_1} + \frac{B_2}{m_2} + \frac{B_3}{m_3} + \frac{B_4}{m_4} = 0$$

From (II), substituting m for x ,

$$B_k \left(1 + \frac{2\delta_2}{m_k} + \frac{\nu_2^2 + \delta_2^2}{m_k^2} \right) = p_{21} A_k$$

$$\therefore \frac{B_1}{m_1^2} + \frac{B_2}{m_2^2} + \frac{B_3}{m_3^2} + \frac{B_4}{m_4^2} = p_{21} \frac{F}{\nu_2^2 + \delta_2^2} = P$$

By the following four equations which are derived from the boundary conditions, we may find constant B .

$$\frac{B_1}{m_1^2} + \frac{B_2}{m_2^2} + \frac{B_3}{m_3^2} + \frac{B_4}{m_4^2} = P$$

$$\frac{B_1}{m_1} + \frac{B_2}{m_2} + \frac{B_3}{m_3} + \frac{B_4}{m_4} = 0$$

$$B_1 + B_2 + B_3 + B_4 = 0$$

$$m_1 B_1 + m_2 B_2 + m_3 B_3 + m_4 B_4 = 0$$

$$B_1 \begin{vmatrix} \frac{1}{m_1^2} & \frac{1}{m_2^2} & \frac{1}{m_3^2} & \frac{1}{m_4^2} \\ \frac{1}{m_1} & \frac{1}{m_2} & \frac{1}{m_3} & \frac{1}{m_4} \\ 1 & 1 & 1 & 1 \\ m_1 & m_2 & m_3 & m_4 \end{vmatrix} = \begin{vmatrix} P & \frac{1}{m_4^2} & \frac{1}{m_3^2} & \frac{1}{m_2^2} \\ 0 & \frac{1}{m_4} & \frac{1}{m_3} & \frac{1}{m_2} \\ 0 & 1 & 1 & 1 \\ 0 & m_4 & m_3 & m_2 \end{vmatrix}$$

$$\therefore \quad B_1 \frac{1}{m_1^2}\frac{1}{m_2^2}\frac{1}{m_3^2}\frac{1}{m_4^2} \begin{vmatrix} 1 & 1 & 1 & 1 \\ m_1 & m_2 & m_3 & m_4 \\ m_1^2 & m_2^2 & m_3^2 & m_4^2 \\ m_1^3 & m_2^3 & m_3^3 & m_4^3 \end{vmatrix} = \frac{1}{m_4^2}\frac{1}{m_3^2}\frac{1}{m_2^2} \begin{vmatrix} P & 1 & 1 & 1 \\ 0 & m_4 & m_3 & m_2 \\ 0 & m_4^2 & m_3^2 & m_2^2 \\ 0 & m_4^3 & m_3^3 & m_2^3 \end{vmatrix}$$

$$B_1 \frac{1}{m_1^2} \begin{vmatrix} 1 & 1 & 1 & 1 \\ m_1 & m_2 & m_3 & m_4 \\ m_1^2 & m_2^2 & m_3^2 & m_4^2 \\ m_1^3 & m_2^3 & m_3^3 & m_4^3 \end{vmatrix} = P \begin{vmatrix} m_4 & m_3 & m_2 \\ m_4^2 & m_3^2 & m_2^2 \\ m_4^3 & m_3^3 & m_2^3 \end{vmatrix}$$

$$= P\, m_4 m_3 m_2 \begin{vmatrix} 1 & 1 & 1 \\ m_4 & m_3 & m_2 \\ m_4^2 & m_3^2 & m_2^2 \end{vmatrix}$$

$$= -P\, m_4 m_3 m_2 \begin{vmatrix} 1 & 1 & 1 \\ m_2 & m_3 & m_4 \\ m_2^2 & m_3^2 & m_4^2 \end{vmatrix}$$

While,

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ m_1 & m_2 & m_3 & m_4 \\ m_1^2 & m_2^2 & m_3^2 & m_4^2 \\ m_1^3 & m_2^3 & m_3^3 & m_4^3 \end{vmatrix} = (m_1 - m_2)(m_1 - m_3)(m_1 - m_4)(m_2 - m_3)(m_2 - m_4)(m_3 - m_4).$$

$$B_1(m_1 - m_2)(m_1 - m_3)(m_1 - m_4) = -P\, m_1^2 m_2 m_3 m_4$$

$$B_2(m_2 - m_3)(m_2 - m_4)(m_2 - m_1) = -P\, m_1 m_2^2 m_3 m_4$$

$$B_3(m_3 - m_4)(m_3 - m_1)(m_3 - m_2) = -P\, m_1 m_2 m_3^2 m_4$$

$$B_4(m_4 - m_1)(m_4 - m_2)(m_4 - m_3) = -P\, m_1 m_2 m_3 m_4^2$$

$$v_2 = e^{-\delta_1 t}\left(B_1 e^{i\nu_1 t} + B_2 e^{-i\nu_1 t}\right] + e^{-\delta_2 t}\left(B_3 e^{i\nu_2 t} + B_4 e^{-i\nu_2 t}\right]$$

$$= A\, e^{-\delta_1 t}\cos(\nu_1 t + c_1) + B\, e^{-\delta_2 t}\cos(\nu_2 t + c_2)$$

Part II Heat Energy of Various Ignition Systems

The opinion is held by many engineers that if an engine runs at good speed, the gas can be ignited by a spark with comparatively low heat quantity. However, at starting of the engine the heat quantity in the spark is an important factor in igniting the mixture.

The American Bureau of Standards has measured the heat in unit sparks for various ignition apparatus of aeronautic engine by a calorimetric method. (Report No.56)

A simplified form of heat measuring apparatus has been used by the writer in studying this problem.

General Principle

We introduce a known constant heat energy into a certain apparatus and note the rate of temperature rise caused by this energy. For this purpose, heat created in a resistance unit by direct current is accurate.

Next, we note how ignition sparks in the same apparatus raise the temperature of that appratus.

Comparing the above two effects, we may compute the heat quantity in a single spark from the following formula.

$$\text{Joules per spark} = \frac{\text{watts} \times 60}{\text{Spark per rev.} \times \text{r.p.m.}}$$

Construction of Calorimeter

The main part of the calorimeter used in this test was made of brass piece set in a double-walled box as shown in Fig.13.

Thin aluminum plate forms the inside wall and sawdust is stuffed between the walls so that we may keep inside temperature uniform.

The difference between this method and that of Bureau of Standards is that the temperature of the brass wall was measured by thermos-couple in the latter, while in the former temperature of cavity in the brass piece was measured by thermometer.

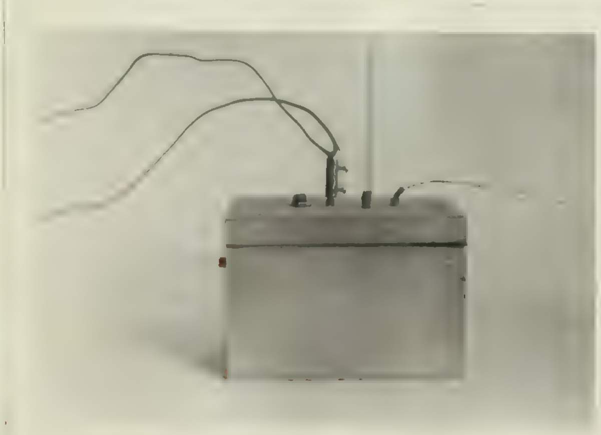
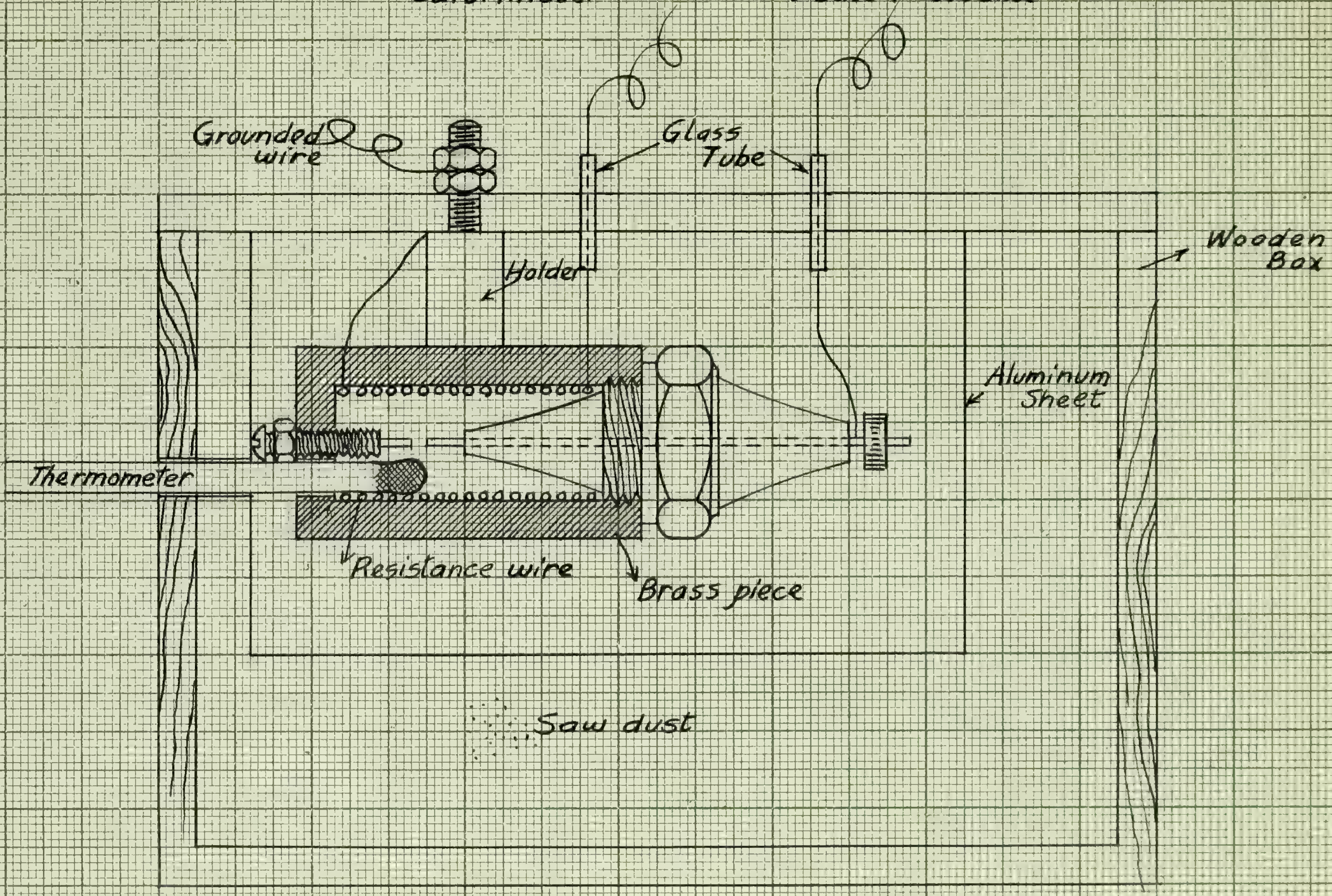


Fig. 13
Calorimeter

Scale: Full size



Process of Experiment

First the calorimeter was calibrated. For this purpose, constant energy was introduced to the cavity for fifteen minutes, readings of the thermometer which measures the temperature in the cavity being taken every minute.

The same initial temperature, 35°C, of the calorimeter was always chosen for beginning either a calibration test or a spark test.

The rise of temperature is irregular during the first three or four seconds.

It is supposed that this comes from the fact that the initial temperature of the cavity is 35°C, while the temperature of the brass piece is not exactly the same as that of the cavity.

To avoid this irregularity, we consider the reading at the fourth second as our zero and get calibration of the apparatus as shown in Fig.13, page 40.

Several experiments were conducted for different ignition systems.

Two different spark gap lengthes were used, $\frac{5}{64}''$ and $\frac{1}{8}''$.

The results of the experiments are shown in Figs.1,2,3, 4, and 5.

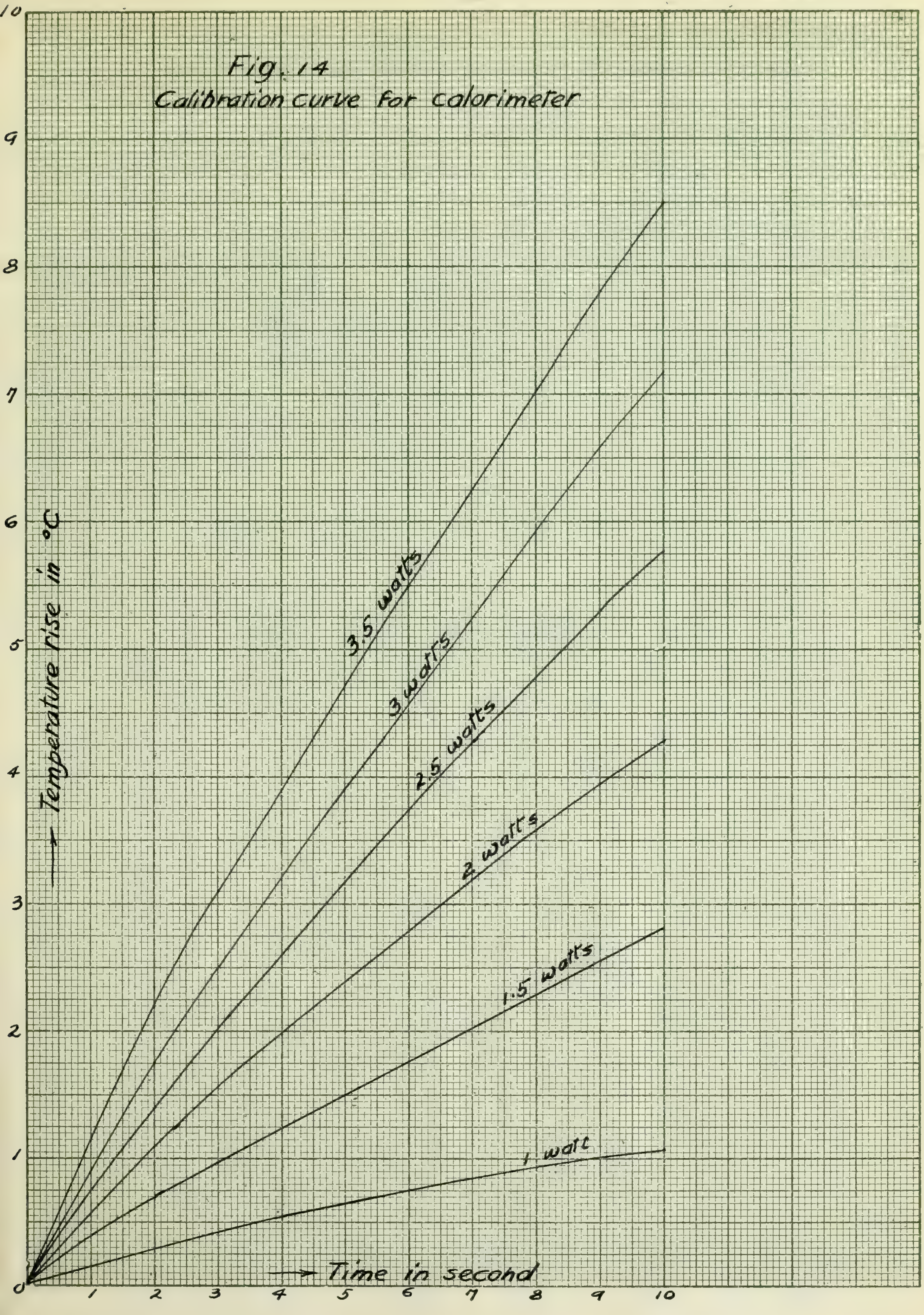


Fig. 14
Calibration curve for calorimeter

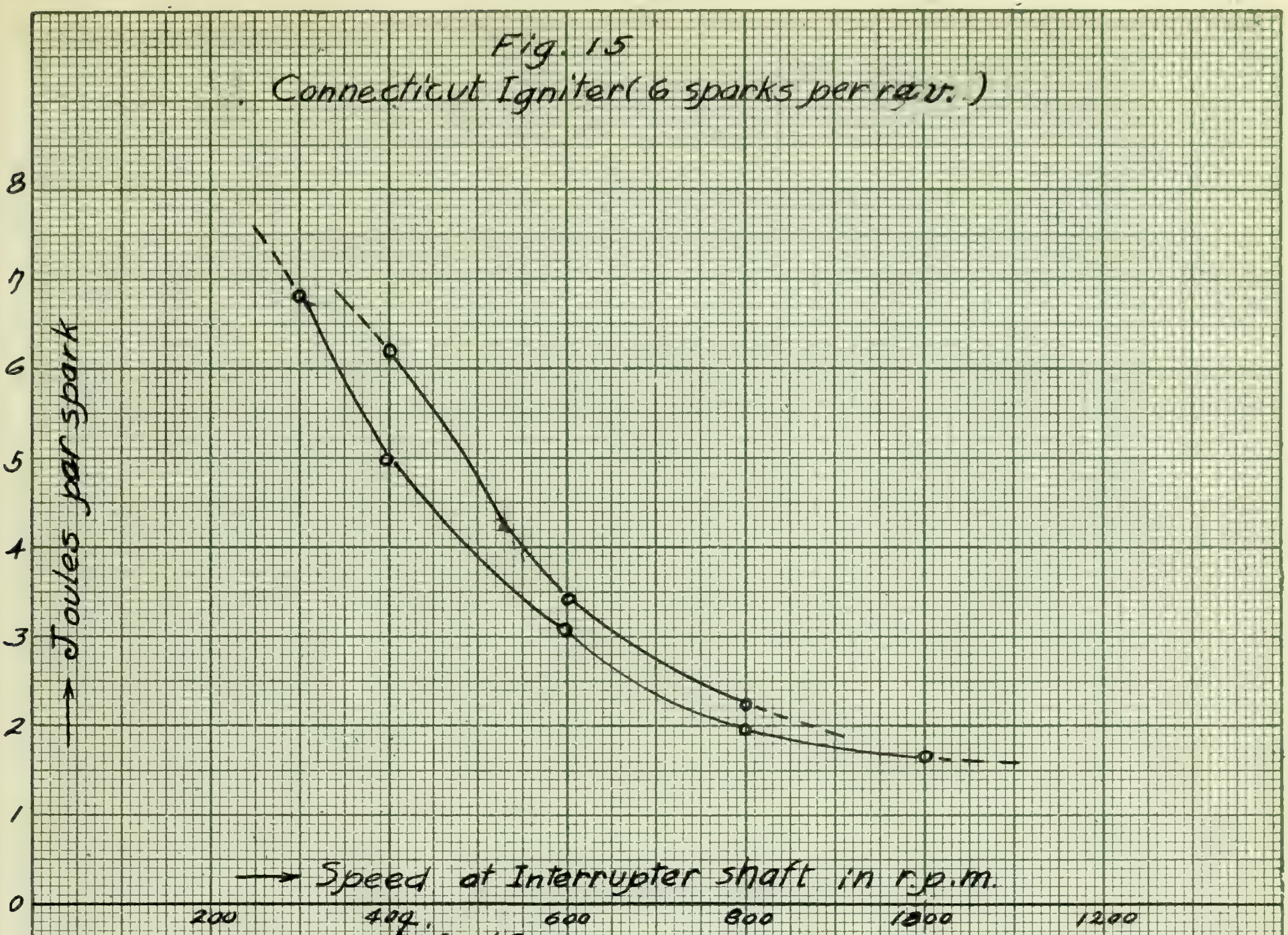
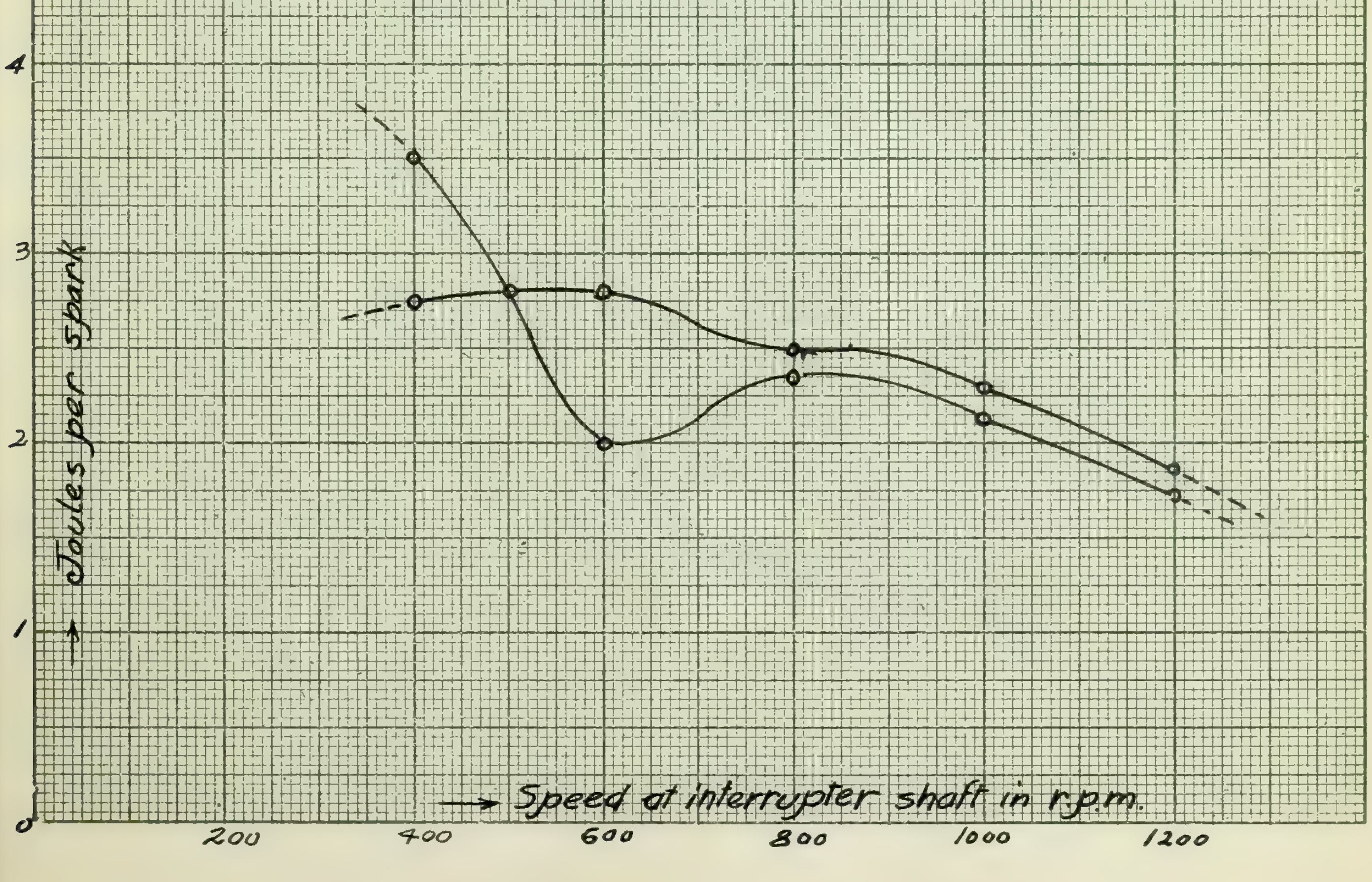


Fig. 15
Connecticut Igniter (6 sparks per rev.)

Fig. 16
Atwater Kent Ignition System (6 sparks per rev.)



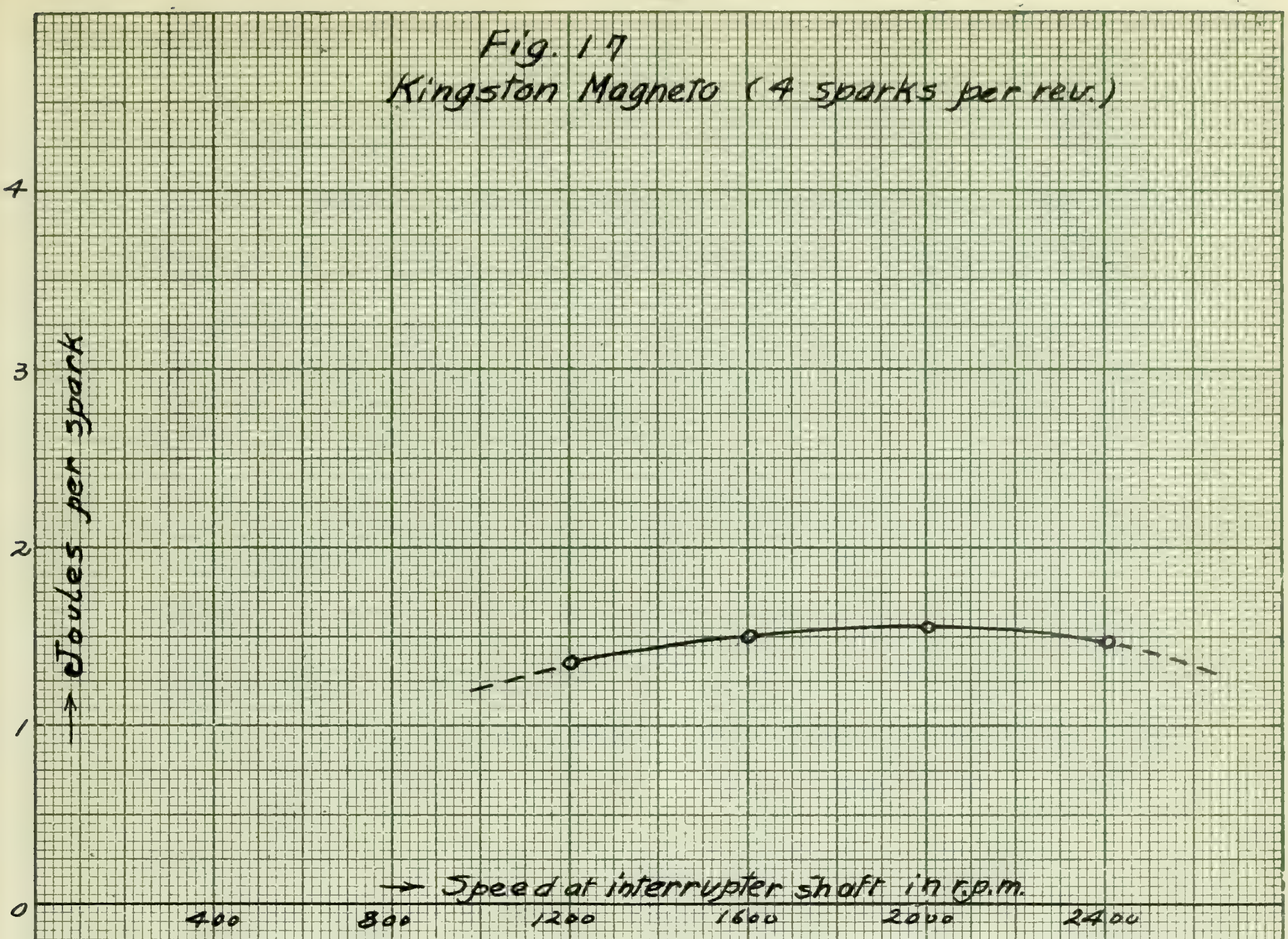


Fig. 17
Kingston Magneto (4 sparks per rev.)

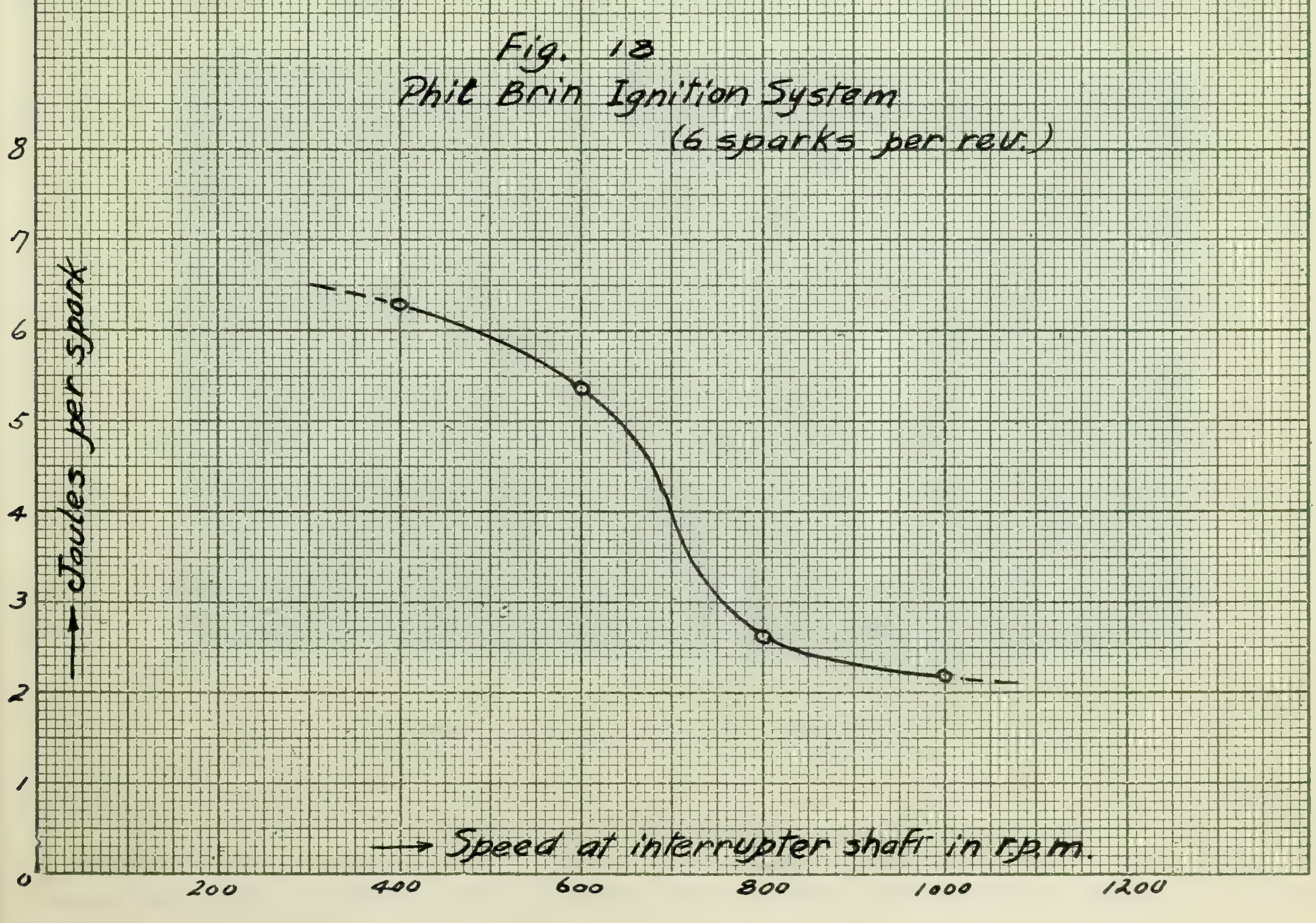


Fig. 18
Phil Brin Ignition System
(6 sparks per rev.)

The energy in unit spark depends greatly upon the primary current at the moment of interruption. Theoretically it is proportional to the square of the current at interruption, because the energy stored in induction coil is

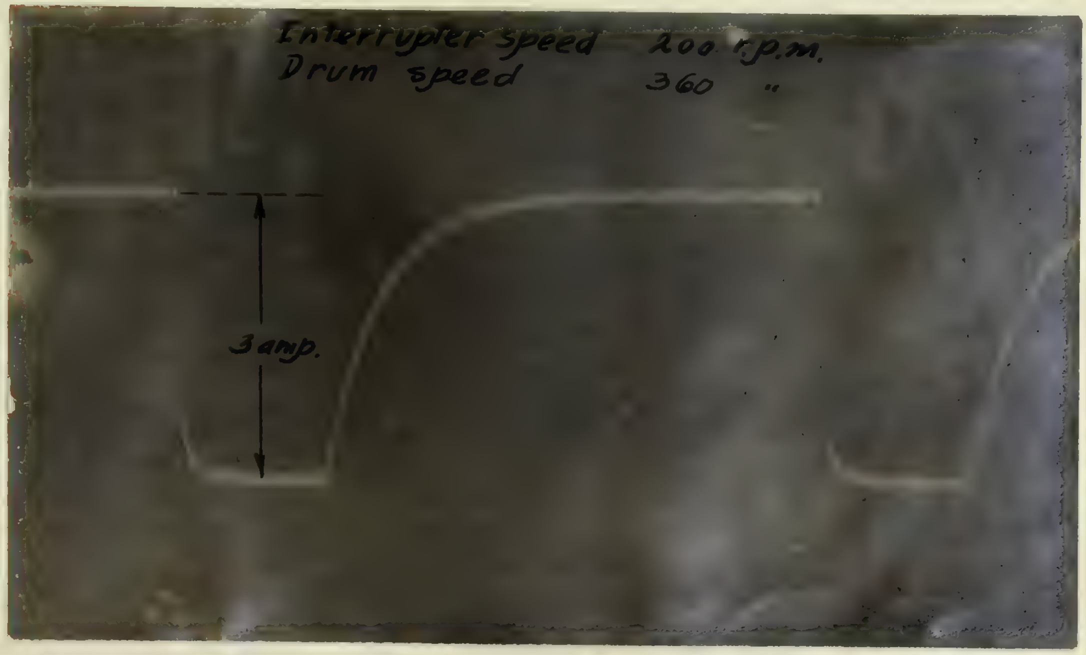
$$W = \frac{L.I}{2}$$

where I : current at interruption.

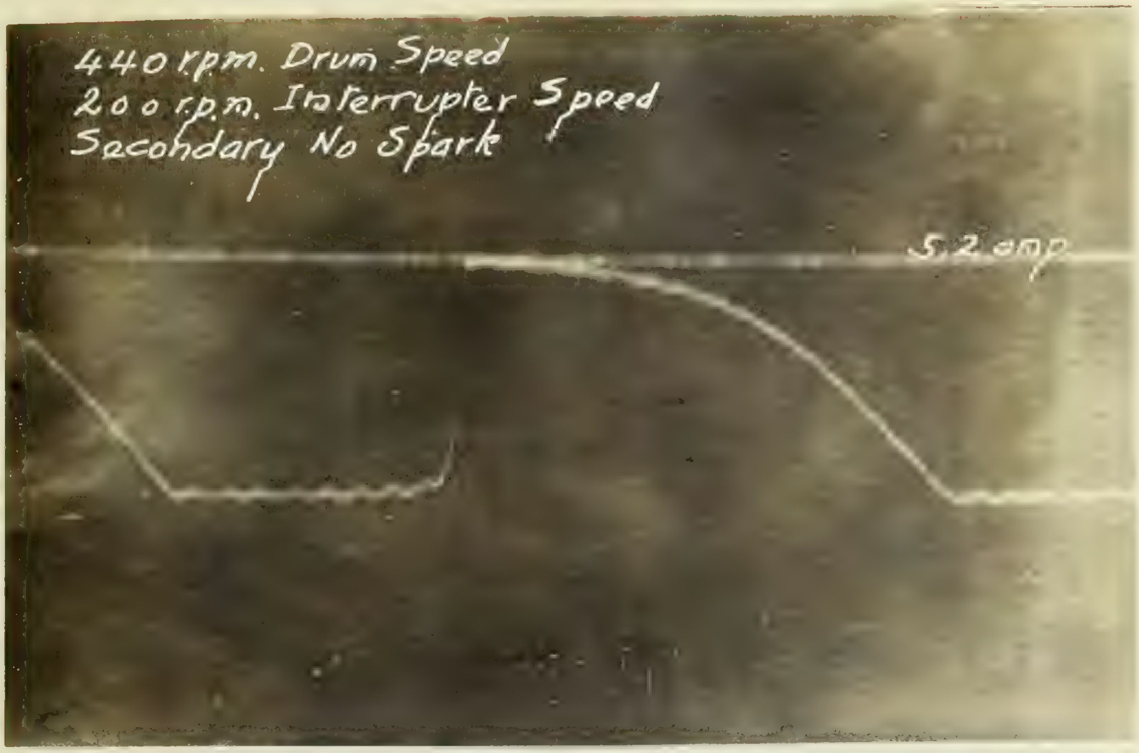
We have already seen that the time to build up the current in the primary circuit is very slow for the Connecticut Igniter, but very fast for the Atwater Kent system.

If we close the primary interrupter permanently, the final current, which depends upon the resistance of the primary coil, is larger in the Connecticut system.

The following oscillograms shows this relation.



Primary current of the Atwater Kent system



Primary current of the Connecticut Igniter

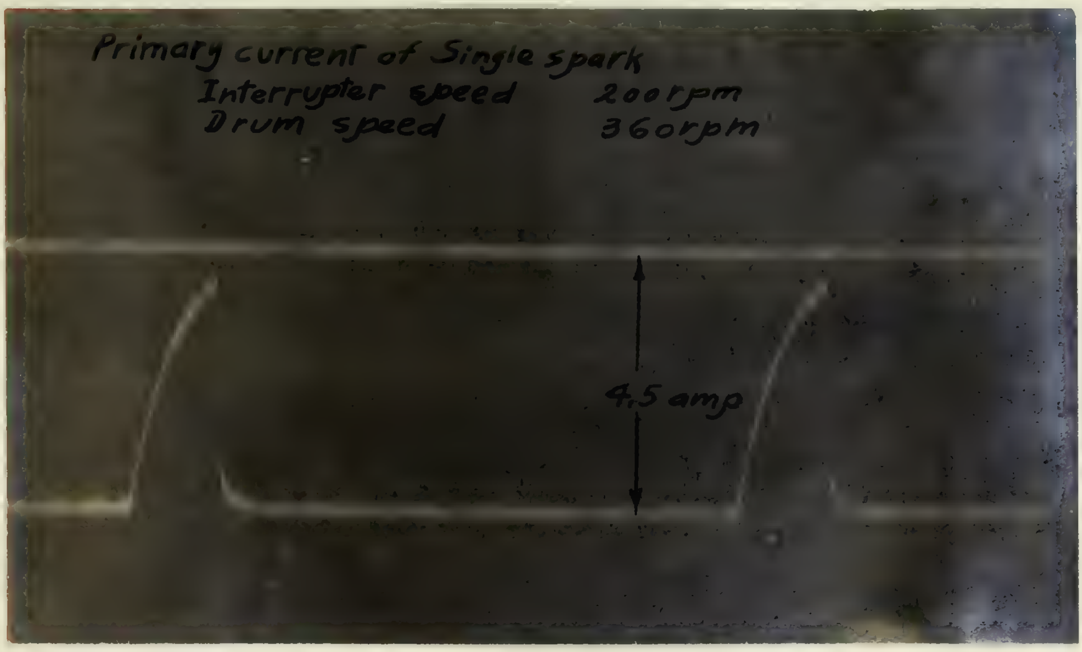
The Connecticut Igniter must have more energy in unit spark at low speeds than has the Atwater Kent. But at higher speeds, near 1000 r.p.m. of interrupter shaft (corresponding to 30 to 40 miles per hour of car speed), the latter yields more energy per spark.

In the Connecticut Igniter, we can get no spark at all at more than 1000 r.p.m. (interrupter shaft) for $\frac{1}{8}''$ spark gap, while in the Atwater Kent system, we may clearly see a spark at 1200 r.p.m. or higher and $\frac{1}{8}''$ spark gap.

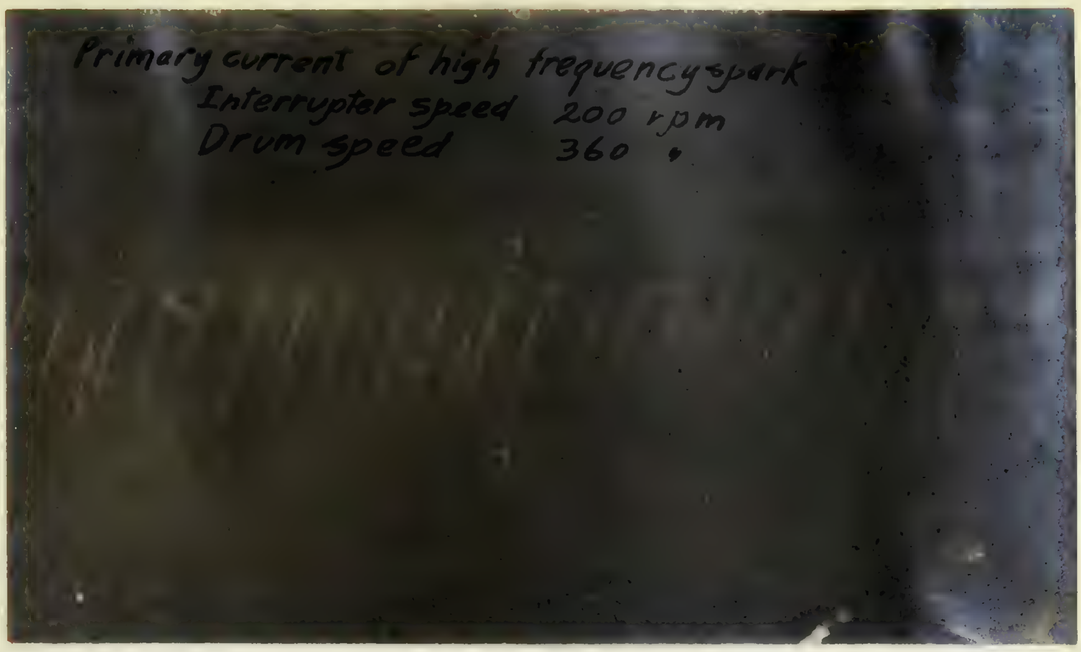
If we change the spark gap length, it seems that at low speed the energy is little larger for the larger gap length and smaller for the smaller gap length.

In the Phil Brin Igniter, the heat energy in unit spark is large with the high frequency system. But the high frequency system seems not to be good for the running condition of the engine, when we need a rather snappy spark.

If we use single spark in this system, the spark energy is so small that we can not measure it accurately by the calorimeter mentioned above.



Single spark of the Phil Brin system



High frequency spark of the Phil Brin system

From the oscillogram for single spark, we see how small is the duration of the interrupter contact.

The primary current of the high frequency spark has the same nature as that of the ordinary induction coil with magnetic interrupter.

Appendex

I

$$E = r_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + \frac{\int i_1 dt}{C_1} \qquad (1)$$

$$0 = r_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + \frac{\int i_2 dt}{C_2} \qquad (2)$$

Differentiating (1) and (2) with respect to t ,

$$0 = r_1 \frac{di_1}{dt} + L_1 \frac{d^2 i_2}{dt^2} + M \frac{d^2 i_2}{dt} + \frac{i_1}{C_1} \qquad (3)$$

$$0 = r_2 \frac{di_2}{dt} + L_2 \frac{d^2 i_2}{dt^2} + M \frac{d^2 i_1}{dt} + \frac{i_2}{C_2} \qquad (4)$$

(3) and (4) are rewritten the following way.

$$0 = \left(L_1 D_t^2 + r_1 D_t + \frac{1}{C_1} \right) i_1 + M D_t^2 \; i_2 \qquad (5)$$

$$0 = M D_t^2 \; i_1 + \left(L_2 D_t^2 + r_2 D_t + \frac{1}{C_2} \right) i_2 \qquad (6)$$

$(5) \times \left(L_2 D_t^2 + r_2 D_t + \frac{1}{C_2} \right)$

$$0 = \left(L_1 D_t^2 + r_1 D_t + \frac{1}{C_1} \right)\left(L_2 D_t^2 + r_2 D_t + \frac{1}{C_2} \right) i_1 + M D_t^2 \left(L_2 D_t^2 + r_2 D_t + \frac{1}{C_2} \right) i_2 \qquad (7)$$

$(6) \times M D_t^2$

$$0 = M^2 D_t^4 \; i_1 + \left(L_2 D_t^2 + r_2 D_t + \frac{1}{C_2} \right) M D_t^2 i_2 \qquad (8)$$

The equation for the primary current: (7) – (8)

$$\begin{aligned} 0 &= \left(L_1 D_t^2 + r_1 D_t + \frac{1}{C_1} \right)\left(L_2 D_t^2 + r_2 D_t + \frac{1}{C_2} \right) i_1 - M^2 D_t^4 i_1 \\ &= \Big\{ (L_1 L_2 - M^2) D_t^4 + (r_1 L_2 + L_1 r_2) D_t^3 + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + r_1 r_2 \right) D_t^2 \\ &\qquad + \left(\frac{r_2}{C_1} + \frac{r_1}{C_2} \right) D_t + \frac{1}{C_1 C_2} \end{aligned} \qquad (9)$$

Similarly, for the secondary current

$$0 = \left\{ (L_1 L_2 - M^2) D_t^4 + (r_1 L_2 - L_1 r_2) D_t^3 + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1} + r_1 r_2 \right) D_t^2 + \left(\frac{r_2}{C_1} + \frac{r_1}{C_2} \right) D_t + \frac{1}{C_1 C_2} \right\} i_2 \qquad (10)$$

II

$$\frac{1}{(E-ir)(1+ki)^2} = \frac{A}{E-ir} + \frac{B}{1+ki} + \frac{C}{(1+ki)^2}$$

$$= \frac{A(1+ki)^2+B(1+ki)(E-ir)+C(E-ir)}{(E-ir)(1+ki)^2}$$

$$\therefore \quad A(1+ki)^2+B(1+ki)(E-ir)+C(E-ir)=1$$

If $i=\frac{E}{r}$, the above equation becomes

$$A\left(1+k\frac{E}{r}\right)^2=1$$

$$\therefore \quad A = \frac{r^2}{(r+kE)^2}$$

If $i=-\frac{1}{k}$,

$$C\left(E+\frac{r}{k}\right)=1$$

$$C = \frac{k}{r+kE}$$

If $i=0$,

$$\frac{r^2}{(r+kE)^2} + EB + \frac{kE}{r+kE} = 1$$

$$EB = 1 - \frac{rkE+k^2E^2+r^2}{(r+kE)^2}$$

$$= \frac{rkE}{(r+kE)^2}$$

$$\therefore \quad B = \frac{rk}{(r+kE)^2}$$

If we substitute $r+kE$ for a, then

$$A=\frac{r^2}{a^2}, \quad B=\frac{rk}{a^2}, \text{ and } C=\frac{k}{a}$$

$$\therefore \quad \frac{1}{(E-ir)(1+ki)^2} = \frac{r^2}{(E-ir)a^2} + \frac{kr}{(1+ki)a^2} + \frac{k}{(1+ki)a}$$

III

$$r_1(A\cos\beta t+B\sin\beta t)-L_1\alpha\,(A\cos\beta t+B\sin\beta t)$$

$$+\beta L_1(-A\sin\beta t+B\cos\beta t)$$

where $A=I$ and

$$B=\frac{(I_0-I)r_1+L_1I\alpha}{\beta\ L_1}$$

$$\therefore\ r_1I\cos\beta t+\frac{(I_0-I)r_1^2+L_1I\alpha r}{\beta\ L_1}\sin\beta t-L_1\alpha I\cos\beta t$$

$$-\frac{\alpha(I_0-I)r_1+L_1I\alpha^2}{\beta}\sin\beta t-\beta L_1I\sin\beta t+I_0r_1\cos\beta t$$

$$-Ir_1\cos\beta t+L_1I\alpha\cos\beta t$$

$$=\frac{(I_0-I)r_1^2}{\beta L_1}\sin\beta t+\frac{I\alpha r}{\beta}\sin\beta t-\frac{\alpha I_0r_1}{\beta}\sin\beta t$$

$$+\frac{\alpha Ir_1}{\beta}\sin\beta t-\frac{L_1I\alpha^2}{\beta}\sin\beta t-\beta L_1I\sin\beta t+I_0r_1\cos\beta t$$

$$=\left\{\frac{(I_0-I)r_1^2}{\beta L_1}-\frac{(\alpha^2+\beta^2)IL}{\beta}-\frac{(I_0-2I)\alpha r_1}{\beta}\right\}\sin\beta t+I_0r_1\cos\beta t$$

Put $\alpha=\frac{r_1}{2L_1}$

$$=\left\{\frac{I_0r_1^2}{2L_1\beta}-\frac{(\alpha^2+\beta^2)IL}{\beta}\right\}\sin\beta t+I_0r_1\cos\beta t$$

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